代幾 I 計算演習 (2005/10/20) の解答

A.1

$$
\begin{aligned}
\left(\begin{pmatrix} -3-i \\ -2+i \\ -3-3i \\ 2-i \\ 2+2i \\ 2-2i \end{pmatrix}, \begin{pmatrix} 2-i \\ -2-3i \\ 0 \\ -1-3i \\ 2-i \\ -3-3i \end{pmatrix}\right) &= (-3-i)\times\overline{(2-i)} + (-2+i)\times\overline{(-2-3i)} \\
&+ (-3-3i)\times\overline{(0)} + (2-i)\times\overline{(-1-3i)} \\
&+ (2+2i)\times\overline{(2-i)} + (2-2i)\times\overline{(-3-3i)} \\
&= (-3-i)\times(2+i) + (-2+i)\times(-2+3i) \\
&+ (-3-3i)\times(0) + (2-i)\times(-1+3i) \\
&+ (2+2i)\times(2+i) + (2-2i)\times(-3+3i) \\
&= (-5-5i) + (1-8i) + (0) + (1+7i) \\
&+ (2+6i) + (12i) \\
&= -1+12i
\end{aligned}
$$

A.2

$$
\begin{aligned}
\left(\begin{pmatrix} -3i \\ -2-3i \\ 2-3i \\ -3+2i \\ 1+i \\ -1+i \end{pmatrix}, \begin{pmatrix} 1+i \\ -2-3i \\ -3+2i \\ -1-i \\ 1-2i \\ 2i \end{pmatrix}\right) &= (-3i)\times\overline{(1+i)} + (-2-3i)\times\overline{(-2-3i)} \\
&+ (2-3i)\times\overline{(-3+2i)} + (-3+2i)\times\overline{(-1-i)} \\
&+ (1+i)\times\overline{(1-2i)} + (-1+i)\times\overline{(2i)} \\
&= (-3i)\times(1-i) + (-2-3i)\times(-2+3i) \\
&+ (2-3i)\times(-3-2i) + (-3+2i)\times(-1+i) \\
&+ (1+i)\times(1+2i) + (-1+i)\times(-2i) \\
&= (-3-3i) + (13) + (-12+5i) + (1-5i) \\
&+ (-1+3i) + (2+2i) \\
&= 2i
\end{aligned}
$$

A.3

$$
\begin{aligned}
\left(\begin{pmatrix} 2-2i \\ -3-3i \\ -3-2i \\ 2i \\ 1-i \end{pmatrix}, \begin{pmatrix} 1+i \\ -3-2i \\ -2-i \\ 2-3i \\ 1-3i \end{pmatrix}\right) &= (2-2i)\times\overline{(1+i)} + (-3-3i)\times\overline{(-3-2i)} \\
&+ (-3-2i)\times\overline{(-2-i)} + (2i)\times\overline{(2-3i)} \\
&+ (1-i)\times\overline{(1-3i)} \\
&= (2-2i)\times(1-i) + (-3-3i)\times(-3+2i) \\
&+ (-3-2i)\times(-2+i) + (2i)\times(2+3i) \\
&+ (1-i)\times(1+3i) \\
&= (-4i) + (15+3i) + (8+i) + (-6+4i) \\
&+ (4+2i) \\
&= 21+6i
\end{aligned}
$$

A.4

$$
\begin{aligned}
\left(\begin{pmatrix} 2-2i \\ 2-2i \\ -2-i \\ 2-2i \\ 2-i \\ -2-2i \end{pmatrix}, \begin{pmatrix} i \\ -1 \\ -3-3i \\ -2 \\ -3 \\ 1-2i \end{pmatrix}\right) &= (2-2i)\times\overline{(i)} + (2-2i)\times\overline{(-1)} \\
&+ (-2-i)\times\overline{(-3-3i)} + (2-2i)\times\overline{(-2)} \\
&+ (2-i)\times\overline{(-3)} + (-2-2i)\times\overline{(1-2i)} \\
&= (2-2i)\times(-i) + (2-2i)\times(-1) \\
&+ (-2-i)\times(-3+3i) + (2-2i)\times(-2) \\
&+ (2-i)\times(-3) + (-2-2i)\times(1+2i) \\
&= (-2-2i) + (-2+2i) + (9-3i) + (-4+4i) \\
&+ (-6+3i) + (2-6i) \\
&= -3-2i
\end{aligned}
$$

A.5

$$
\begin{aligned}
(\begin{pmatrix} 2 \\ -2 \\ -2-i \\ -3+i \\ -3 \\ 1+2i \end{pmatrix}, \begin{pmatrix} 0 \\ -3+2i \\ 2i \\ -1+2i \\ 2-i \\ -3i \end{pmatrix}) &= (2) \times \overline{(0)} + (-2) \times \overline{(-3+2i)} \\
&+ (-2-i) \times \overline{(2i)} + (-3+i) \times \overline{(-1+2i)} \\
&+ (-3) \times \overline{(2-i)} + (1+2i) \times \overline{(-3i)} \\
&= (2) \times (0) + (-2) \times (-3-2i) \\
&+ (-2-i) \times (-2i) + (-3+i) \times (-1-2i) \\
&+ (-3) \times (2+i) + (1+2i) \times (3i) \\
&= (0) + (6+4i) + (-2+4i) + (5+5i) \\
&+ (-6-3i) + (-6+3i) \\
&= -3+13i
\end{aligned}
$$

A.6

$$
\begin{aligned}
(\begin{pmatrix} -1-2i \\ -i \\ 2+i \\ 1-3i \\ 0 \\ -1-2i \end{pmatrix}, \begin{pmatrix} -2 \\ 1+i \\ -2-3i \\ 2-i \\ -i \\ 1-i \end{pmatrix}) &= (-1-2i) \times \overline{(-2)} + (-i) \times \overline{(1+i)} \\
&+ (2+i) \times \overline{(-2-3i)} + (1-3i) \times \overline{(2-i)} \\
&+ (0) \times \overline{(-i)} + (-1-2i) \times \overline{(1-i)} \\
&= (-1-2i) \times (-2) + (-i) \times (1-i) \\
&+ (2+i) \times (-2+3i) + (1-3i) \times (2+i) \\
&+ (0) \times (i) + (-1-2i) \times (1+i) \\
&= (2+4i) + (-1-i) + (-7+4i) + (5-5i) \\
&+ (0) + (1-3i) \\
&= -i
\end{aligned}
$$

A.7

$$
\begin{aligned}
\left(\begin{pmatrix} 0 \\ -2-3i \\ 1+2i \\ 2+i \\ -3+i \\ -2+i \end{pmatrix}, \begin{pmatrix} -1-2i \\ -i \\ 2-i \\ -2 \\ -3-2i \\ 1+2i \end{pmatrix}\right) &= (0) \times \overline{(-1-2i)} + (-2-3i) \times \overline{(-i)} \\
&+ (1+2i) \times \overline{(2-i)} + (2+i) \times \overline{(-2)} \\
&+ (-3+i) \times \overline{(-3-2i)} + (-2+i) \times \overline{(1+2i)} \\
&= (0) \times (-1+2i) + (-2-3i) \times (i) \\
&+ (1+2i) \times (2+i) + (2+i) \times (-2) \\
&+ (-3+i) \times (-3+2i) + (-2+i) \times (1-2i) \\
&= (0) + (3-2i) + (5i) + (-4-2i) \\
&+ (7-9i) + (5i) \\
&= 6-3i
\end{aligned}
$$

A.8

$$
\begin{aligned}
\left(\begin{pmatrix} -2-i \\ -2+i \\ -2i \\ -3+i \\ -3-3i \\ -3-3i \end{pmatrix}, \begin{pmatrix} 1-3i \\ -2+2i \\ -1+2i \\ 2i \\ 1-3i \\ -3 \end{pmatrix}\right) &= (-2-i) \times \overline{(1-3i)} + (-2+i) \times \overline{(-2+2i)} \\
&+ (-2i) \times \overline{(-1+2i)} + (-3+i) \times \overline{(2i)} \\
&+ (-3-3i) \times \overline{(1-3i)} + (-3-3i) \times \overline{(-3)} \\
&= (-2-i) \times (1+3i) + (-2+i) \times (-2-2i) \\
&+ (-2i) \times (-1-2i) + (-3+i) \times (-2i) \\
&+ (-3-3i) \times (1+3i) + (-3-3i) \times (-3) \\
&= (1-7i) + (6+2i) + (-4+2i) + (2+6i) \\
&+ (6-12i) + (9+9i) \\
&= 20
\end{aligned}
$$

A.9

$$
\begin{aligned}
\left(\begin{pmatrix} -1-2i \\ i \\ 1+2i \\ 1+i \\ 2 \end{pmatrix}, \begin{pmatrix} -3+2i \\ -1 \\ -i \\ 1+2i \\ -1-i \end{pmatrix}\right) &= (-1-2i) \times \overline{(-3+2i)} + (i) \times \overline{(-1)} \\
&+ (1+2i) \times \overline{(-i)} + (1+i) \times \overline{(1+2i)} \\
&+ (2) \times \overline{(-1-i)} \\
&= (-1-2i) \times (-3-2i) + (i) \times (-1) \\
&+ (1+2i) \times (i) + (1+i) \times (1-2i) \\
&+ (2) \times (-1+i) \\
&= (-1+8i) + (-i) + (-2+i) + (3-i) \\
&+ (-2+2i) \\
&= -2+9i
\end{aligned}
$$

A.10

$$
\begin{aligned}
\left(\begin{pmatrix} 2+i \\ -3 \\ -3 \\ -3 \\ -2-3i \end{pmatrix}, \begin{pmatrix} 1+i \\ i \\ -2-i \\ -1-3i \\ -3+2i \end{pmatrix}\right) &= (2+i) \times \overline{(1+i)} + (-3) \times \overline{(i)} \\
&+ (-3) \times \overline{(-2-i)} + (-3) \times \overline{(-1-3i)} \\
&+ (-2-3i) \times \overline{(-3+2i)} \\
&= (2+i) \times (1-i) + (-3) \times (-i) \\
&+ (-3) \times (-2+i) + (-3) \times (-1+3i) \\
&+ (-2-3i) \times (-3-2i) \\
&= (3-i) + (3i) + (6-3i) + (3-9i) \\
&+ (13i) \\
&= 12+3i
\end{aligned}
$$