

代幾 I 計算演習 (2005/12/01) の解答

1 連立方程式

A.1

$$\begin{cases} x_0 = -6 \\ x_1 = -7 \\ x_2 = -6 \\ x_3 = -7 \\ x_4 = -7 \\ x_5 = -1 \end{cases}$$

単一

A.2

$$\begin{cases} x_0 = -9 \\ x_1 = 5 \\ x_2 = 3p_0 - 3 \\ x_3 = -6 \\ x_4 = p_0 \end{cases}$$

不定

A.3

$$\begin{cases} x_0 + x_4 = -5 \\ x_1 - 2x_4 = 7 \\ x_2 = -1 \\ x_3 - 3x_4 = 1 \\ 0 = 1 \\ 0 = 0 \end{cases}$$

不能

2 逆行列

A.4

$$\begin{aligned} & \left(\begin{array}{ccc|ccc} 4 & 6 & 5 & 1 & 0 & 0 \\ 7 & 1 & 0 & 0 & 1 & 0 \\ 3 & 4 & 5 & 0 & 0 & 1 \end{array} \right) \xrightarrow{(0) = \frac{1}{4} \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{5}{4} & \frac{1}{4} & 0 & 0 \\ 7 & 1 & 0 & 0 & 1 & 0 \\ 3 & 4 & 5 & 0 & 0 & 1 \end{array} \right) \\ & \xrightarrow{(1) = (1) - 7 \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{5}{4} & \frac{1}{4} & 0 & 0 \\ 0 & -\frac{19}{2} & -\frac{35}{4} & -\frac{7}{4} & 1 & 0 \\ 3 & 4 & 5 & 0 & 0 & 1 \end{array} \right) \end{aligned}$$

$$\begin{aligned}
& \xrightarrow{(2) = (2) - 3 \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{5}{4} & \frac{1}{4} & 0 & 0 \\ 0 & -\frac{19}{2} & -\frac{35}{4} & -\frac{7}{4} & 1 & 0 \\ 0 & -\frac{1}{2} & \frac{5}{4} & -\frac{3}{4} & 0 & 1 \end{array} \right) \\
& \xrightarrow{(1) = -\frac{2}{19} \times (1)} \left(\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{5}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{35}{38} & \frac{7}{38} & -\frac{2}{19} & 0 \\ 0 & -\frac{1}{2} & \frac{5}{4} & -\frac{3}{4} & 0 & 1 \end{array} \right) \\
& \xrightarrow{(2) = (2) + \frac{1}{2} \times (1)} \left(\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{5}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{35}{38} & \frac{7}{38} & -\frac{2}{19} & 0 \\ 0 & 0 & \frac{65}{38} & -\frac{25}{38} & -\frac{1}{19} & 1 \end{array} \right) \\
& \xrightarrow{(2) = \frac{38}{65} \times (2)} \left(\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{5}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{35}{38} & \frac{7}{38} & -\frac{2}{19} & 0 \\ 0 & 0 & 1 & -\frac{5}{13} & -\frac{2}{65} & \frac{38}{65} \end{array} \right) \\
& \xrightarrow{(0) = (0) - \frac{3}{2} \times (1)} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{5}{38} & -\frac{1}{38} & \frac{3}{19} & 0 \\ 0 & 1 & \frac{35}{38} & \frac{7}{38} & -\frac{2}{19} & 0 \\ 0 & 0 & 1 & -\frac{5}{13} & -\frac{2}{65} & \frac{38}{65} \end{array} \right) \\
& \xrightarrow{(0) = (0) + \frac{5}{38} \times (2)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{13} & \frac{2}{13} & \frac{1}{13} \\ 0 & 1 & \frac{35}{38} & \frac{7}{38} & -\frac{2}{19} & 0 \\ 0 & 0 & 1 & -\frac{5}{13} & -\frac{2}{65} & \frac{38}{65} \end{array} \right) \\
& \xrightarrow{(1) = (1) - \frac{35}{38} \times (2)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{13} & \frac{2}{13} & \frac{1}{13} \\ 0 & 1 & 0 & \frac{7}{13} & -\frac{1}{13} & -\frac{7}{13} \\ 0 & 0 & 1 & -\frac{5}{13} & -\frac{2}{65} & \frac{38}{65} \end{array} \right) \\
& \left(\begin{array}{ccc|ccc} -\frac{1}{13} & \frac{2}{13} & \frac{1}{13} & 4 & 6 & 5 \\ \frac{7}{13} & -\frac{1}{13} & -\frac{7}{13} & 7 & 1 & 0 \\ -\frac{5}{13} & -\frac{2}{65} & \frac{38}{65} & 3 & 4 & 5 \end{array} \right) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right)
\end{aligned}$$

A.5

$$\begin{aligned}
& \left(\begin{array}{ccc|ccc} 8 & 4 & 4 & 1 & 0 & 0 \\ 6 & 3 & 5 & 0 & 1 & 0 \\ 2 & 1 & 7 & 0 & 0 & 1 \end{array} \right) \xrightarrow{(0) = \frac{1}{8} \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{8} & 0 & 0 \\ 6 & 3 & 5 & 0 & 1 & 0 \\ 2 & 1 & 7 & 0 & 0 & 1 \end{array} \right) \\
& \xrightarrow{(1) = (1) - 6 \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 2 & -\frac{3}{4} & 1 & 0 \\ 2 & 1 & 7 & 0 & 0 & 1 \end{array} \right) \\
& \xrightarrow{(2) = (2) - 2 \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 2 & -\frac{3}{4} & 1 & 0 \\ 0 & 0 & 6 & -\frac{1}{4} & 0 & 1 \end{array} \right) \\
& \xrightarrow{(2) = \frac{1}{6} \times (2)} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 2 & -\frac{3}{4} & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{24} & 0 & \frac{1}{6} \end{array} \right)
\end{aligned}$$

$$\begin{aligned}
& \xrightarrow{\quad} (0) = (0) - \frac{1}{2} \times (2) \quad \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & \frac{7}{48} & 0 & -\frac{1}{12} \\ 0 & 0 & 2 & -\frac{3}{4} & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{24} & 0 & \frac{1}{6} \end{array} \right) \\
& \xrightarrow{\quad} (1) = (1) - 2 \times (2) \quad \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & \frac{7}{48} & 0 & -\frac{1}{12} \\ 0 & 0 & 0 & -\frac{2}{3} & 1 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{24} & 0 & \frac{1}{6} \end{array} \right) \\
\left(\begin{array}{ccc|ccc} \frac{7}{48} & 0 & -\frac{1}{12} & 8 & 4 & 4 \\ -\frac{2}{3} & 1 & -\frac{1}{3} & 6 & 3 & 5 \\ -\frac{1}{24} & 0 & \frac{1}{6} & 2 & 1 & 7 \end{array} \right) & = \quad \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & & & \\ 0 & 0 & 0 & & & \\ 0 & 0 & 1 & & & \end{array} \right)
\end{aligned}$$

A.6

$$\begin{aligned}
& \left(\begin{array}{ccc|ccc} 3 & 2 & 6 & 1 & 0 & 0 \\ 4 & 6 & 6 & 0 & 1 & 0 \\ 5 & 1 & 7 & 0 & 0 & 1 \end{array} \right) \xrightarrow{(0) = \frac{1}{3} \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{2}{3} & 2 & \frac{1}{3} & 0 & 0 \\ 4 & 6 & 6 & 0 & 1 & 0 \\ 5 & 1 & 7 & 0 & 0 & 1 \end{array} \right) \\
& \xrightarrow{(1) = (1) - 4 \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{2}{3} & 2 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{10}{3} & -2 & -\frac{4}{3} & 1 & 0 \\ 5 & 1 & 7 & 0 & 0 & 1 \end{array} \right) \\
& \xrightarrow{(2) = (2) - 5 \times (0)} \left(\begin{array}{ccc|ccc} 1 & \frac{2}{3} & 2 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{10}{3} & -2 & -\frac{4}{3} & 1 & 0 \\ 0 & -\frac{7}{3} & -3 & -\frac{5}{3} & 0 & 1 \end{array} \right) \\
& \xrightarrow{(1) = \frac{3}{10} \times (1)} \left(\begin{array}{ccc|ccc} 1 & \frac{2}{3} & 2 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{5} & -\frac{2}{5} & \frac{3}{10} & 0 \\ 0 & -\frac{7}{3} & -3 & -\frac{5}{3} & 0 & 1 \end{array} \right) \\
& \xrightarrow{(2) = (2) + \frac{7}{3} \times (1)} \left(\begin{array}{ccc|ccc} 1 & \frac{2}{3} & 2 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{5} & -\frac{2}{5} & \frac{3}{10} & 0 \\ 0 & 0 & -\frac{22}{5} & -\frac{13}{5} & \frac{7}{10} & 1 \end{array} \right) \\
& \xrightarrow{(2) = -\frac{5}{22} \times (2)} \left(\begin{array}{ccc|ccc} 1 & \frac{2}{3} & 2 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{5} & -\frac{2}{5} & \frac{3}{10} & 0 \\ 0 & 0 & 1 & \frac{13}{22} & -\frac{7}{44} & -\frac{5}{22} \end{array} \right) \\
& \xrightarrow{(0) = (0) - \frac{2}{3} \times (1)} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{12}{5} & \frac{3}{5} & -\frac{1}{5} & 0 \\ 0 & 1 & -\frac{3}{5} & -\frac{2}{5} & \frac{3}{10} & 0 \\ 0 & 0 & 1 & \frac{13}{22} & -\frac{7}{44} & -\frac{5}{22} \end{array} \right) \\
& \xrightarrow{(0) = (0) - \frac{12}{5} \times (2)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{9}{11} & \frac{2}{11} & \frac{6}{11} \\ 0 & 1 & -\frac{3}{5} & -\frac{2}{5} & \frac{3}{10} & 0 \\ 0 & 0 & 1 & \frac{13}{22} & -\frac{7}{44} & -\frac{5}{22} \end{array} \right) \\
& \xrightarrow{(1) = (1) + \frac{3}{5} \times (2)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{9}{11} & \frac{2}{11} & \frac{6}{11} \\ 0 & 1 & 0 & -\frac{1}{22} & \frac{9}{44} & -\frac{3}{22} \\ 0 & 0 & 1 & \frac{13}{22} & -\frac{7}{44} & -\frac{5}{22} \end{array} \right)
\end{aligned}$$

$$\begin{pmatrix} -\frac{9}{11} & \frac{2}{11} & \frac{6}{11} \\ -\frac{1}{22} & \frac{9}{44} & -\frac{3}{22} \\ \frac{13}{22} & -\frac{7}{44} & -\frac{5}{22} \end{pmatrix} \begin{pmatrix} 3 & 2 & 6 \\ 4 & 6 & 6 \\ 5 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3 行列式

A.7

$$\begin{aligned} \begin{vmatrix} 1 & 1 & -1 & -3 \\ -3 & -2 & 3 & 5 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{vmatrix} &= \begin{vmatrix} 3 & 0 & 0 & 0 \\ 5 & -2 & 3 & -3 \\ 0 & -1 & 0 & 2 \\ -3 & 1 & -1 & 1 \end{vmatrix} & 4,4 \\ &= 3 \begin{vmatrix} -2 & 3 & -3 \\ -1 & 0 & 2 \\ 1 & -1 & 1 \end{vmatrix} \\ &= 6 \end{aligned}$$

A.8

$$\begin{aligned} \begin{vmatrix} -1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ -2 & -2 & 1 & 0 \\ 2 & -1 & -1 & 1 \end{vmatrix} &= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ -2 & -2 & 1 & 0 \\ -1 & 2 & -1 & 1 \end{vmatrix} & 2,2 \\ &= \begin{vmatrix} -1 & 1 & -1 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{vmatrix} \\ &= 1 \end{aligned}$$

A.9

$$\begin{aligned} \begin{vmatrix} 2 & -2 & 0 & 0 \\ -2 & -5 & 5 & -7 \\ 0 & 2 & 0 & 0 \\ 0 & -3 & -1 & 2 \end{vmatrix} &= \begin{vmatrix} 2 & 0 & 0 & 0 \\ -5 & -2 & 5 & -7 \\ -2 & 2 & 0 & 0 \\ -3 & 0 & -1 & 2 \end{vmatrix} & 3,2 \\ &= 2 \begin{vmatrix} -2 & 5 & -7 \\ 2 & 0 & 0 \\ 0 & -1 & 2 \end{vmatrix} \\ &= -2 \begin{vmatrix} 2 & 0 & 0 \\ -2 & 5 & -7 \\ 0 & -1 & 2 \end{vmatrix} & 2,1 \\ &= -4 \begin{vmatrix} 5 & -7 \\ -1 & 2 \end{vmatrix} \\ &= -12 \end{aligned}$$

A.10

$$\begin{aligned} &= \begin{vmatrix} -2 & 2 \\ -5 & -4 \end{vmatrix} \times \begin{vmatrix} 0 & 1 & -2 \\ 0 & 1 & -3 \\ 2 & -2 & 2 \end{vmatrix} \\ &= 18 \times -2 \\ &= -36 \end{aligned}$$

A.11

$$\begin{aligned} &= \begin{vmatrix} -7 \end{vmatrix} \times \begin{vmatrix} 2 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & 2 & -3 \end{vmatrix} \times \begin{vmatrix} 8 \end{vmatrix} \\ &= -7 \times 4 \times 8 \\ &= -224 \end{aligned}$$