

## 代幾 I 計算演習 [問題] (2006/05/18)

問. 次の二つの多項式の最大公約数を求めなさい。

Q.1

$$x^5 - 2x^4 - x^3 + 3x^2 + x - 2, \quad x^3 - x^2 - x + 1$$

Q.2

$$x^5 - 4x^4 + 6x^3 - 5x^2 + 3x - 1, \quad x^4 - 2x^3 + x^2$$

Q.3

$$x^3 - x^2, \quad x^2 - 2x + 1$$

Q.4

$$x^5 + 2x^4 + 2x^3 - x^2 - 2x - 2, \quad x^2 - 1$$

Q.5

$$x^6 - 4x^5 + x^4 + 15x^3 - 14x^2 - 13x + 10, \quad x^5 - 3x^4 - 3x^3 + 13x^2 + 5x - 7$$

Q.6

$$x^4 + 2x^3 - 2x^2 - 2x + 4, \quad x^3 + 3x^2 + x - 2$$

Q.7

$$x^4 + 2x^3 + 3x^2 + 2x + 1, \quad x^2 + x + 1$$

Q.8

$$x^7 - 5x^6 + 8x^5 - 2x^4 - 10x^3 + 9x^2 + x - 5, \quad x^5 - 3x^4 + 4x^3 - x^2 - 2x + 2$$

Q.9

$$x^5 + 2x^4 + x^3 - x - 1, \quad x^4 + 3x^3 + 3x^2 + 2x + 1$$

Q.10

$$x^4 + x^3 - x^2 - 4x - 2, \quad x^3 - 2x - 1$$

Q.11

$$x^6 - x^5 - x^4 + 8x^2 - 3x - 6, \quad x^5 - 2x^4 + 9x - 9$$

Q.12

$$x^7 + x^6 - 2x^5 - x^4 + 3x^3 + 3x^2 - 4x + 3, \quad x^6 - 3x^4 + 3x^3 + 2x^2 - 3x + 2$$

Q.13

$$x^5 - 4x^4 + 6x^3 - 7x + 4, \quad x^4 - 3x^3 + 2x^2 + 3x - 3$$

Q.14

$$x^6 - 2x^5 + 4x^3 - 7x^2 + 4x - 3, \quad x^5 - x^4 - 2x^3 + 2x^2 - 2x + 1$$

Q.15

$$x^6 + 4x^5 + 3x^4 - 8x^3 - 9x^2 - 12x - 11, \quad x^5 + 2x^4 - 2x^3 - 3x^2 - 3x - 5$$

Q.16

$$x^6 + 3x^5 + 5x^4 + 7x^3 + 6x^2 + 4x + 2, \quad x^5 + 2x^4 + 3x^3 + 3x^2 + 2x + 1$$

Q.17

$$x^4 + 3x^3 + 4x^2 + 7x + 6, \quad x^3 + 2x^2 + 2x + 4$$

Q.18

$$x^7 + x^6 + x^5 + x^4 + 2x^3 - x - 1, \quad x^6 + x^2 - x$$

Q.19

$$x^4 + x, \quad x^3 + x^2 + x + 1$$

Q.20

$$x^5 - 2x^4 - 3x^3 + 8x^2 - 6, \quad x^4 - x^3 - 4x^2 + 3x + 3$$

Q.21

$$x^4 - 2x^2 + 1, \quad x^2 - 1$$

## 代幾 I 計算演習 [解答] (2006/05/18)

A.1

[問題]  $x^5 - 2x^4 - x^3 + 3x^2 + x - 2, \quad x^3 - x^2 - x + 1$

[計算] 
$$\begin{aligned} x^5 - 2x^4 - x^3 + 3x^2 + x - 2 &= (x^3 - x^2 - x + 1)(x^2 - x - 1) + (x - 1) \\ x^3 - x^2 - x + 1 &= (x - 1)(x^2 - 1) + 0 \end{aligned}$$

[解答]  $x - 1$

A.2

[問題]  $x^5 - 4x^4 + 6x^3 - 5x^2 + 3x - 1, \quad x^4 - 2x^3 + x^2$

[計算] 
$$\begin{aligned} x^5 - 4x^4 + 6x^3 - 5x^2 + 3x - 1 &= (x^4 - 2x^3 + x^2)(x - 2) + (x^3 - 3x^2 + 3x - 1) \\ x^4 - 2x^3 + x^2 &= (x^3 - 3x^2 + 3x - 1)(x + 1) + (x^2 - 2x + 1) \\ x^3 - 3x^2 + 3x - 1 &= (x^2 - 2x + 1)(x - 1) + 0 \end{aligned}$$

[解答]  $x^2 - 2x + 1$

A.3

[問題]  $x^3 - x^2, \quad x^2 - 2x + 1$

[計算] 
$$\begin{aligned} x^3 - x^2 &= (x^2 - 2x + 1)(x + 1) + (x - 1) \\ x^2 - 2x + 1 &= (x - 1)(x - 1) + 0 \end{aligned}$$

[解答]  $x - 1$

A.4

[問題]  $x^5 + 2x^4 + 2x^3 - x^2 - 2x - 2, \quad x^2 - 1$

[計算] 
$$\begin{aligned} x^5 + 2x^4 + 2x^3 - x^2 - 2x - 2 &= (x^2 - 1)(x^3 + 2x^2 + 3x + 1) + (x - 1) \\ x^2 - 1 &= (x - 1)(x + 1) + 0 \end{aligned}$$

[解答]  $x - 1$

A.5

[問題]  $x^6 - 4x^5 + x^4 + 15x^3 - 14x^2 - 13x + 10, \quad x^5 - 3x^4 - 3x^3 + 13x^2 + 5x - 7$

$$\begin{aligned} x^6 - 4x^5 + x^4 + 15x^3 - 14x^2 - 13x + 10 &= (x^5 - 3x^4 - 3x^3 + 13x^2 + 5x - 7)(x - 1) \\ &+ (x^4 - x^3 - 6x^2 - x + 3) \\ \text{[計算]} \quad x^5 - 3x^4 - 3x^3 + 13x^2 + 5x - 7 &= (x^4 - x^3 - 6x^2 - x + 3)(x - 2) \\ &+ (x^3 + 2x^2 - 1) \\ x^4 - x^3 - 6x^2 - x + 3 &= (x^3 + 2x^2 - 1)(x - 3) \\ &+ 0 \end{aligned}$$

[解答]  $x^3 + 2x^2 - 1$

A.6

[問題]  $x^4 + 2x^3 - 2x^2 - 2x + 4, \quad x^3 + 3x^2 + x - 2$

$$\begin{aligned} \text{[計算]} \quad x^4 + 2x^3 - 2x^2 - 2x + 4 &= (x^3 + 3x^2 + x - 2)(x - 1) + (x + 2) \\ x^3 + 3x^2 + x - 2 &= (x + 2)(x^2 + x - 1) + 0 \end{aligned}$$

[解答]  $x + 2$

A.7

[問題]  $x^4 + 2x^3 + 3x^2 + 2x + 1, \quad x^2 + x + 1$

$$\text{[計算]} \quad x^4 + 2x^3 + 3x^2 + 2x + 1 = (x^2 + x + 1)(x^2 + x + 1) + 0$$

[解答]  $x^2 + x + 1$

A.8

[問題]  $x^7 - 5x^6 + 8x^5 - 2x^4 - 10x^3 + 9x^2 + x - 5, \quad x^5 - 3x^4 + 4x^3 - x^2 - 2x + 2$

$$\begin{aligned} x^7 - 5x^6 + 8x^5 - 2x^4 - 10x^3 + 9x^2 + x - 5 &= (x^5 - 3x^4 + 4x^3 - x^2 - 2x + 2)(x^2 - 2x - 2) \\ &+ (x^4 - 2x^3 + x^2 + x - 1) \\ \text{[計算]} \quad x^5 - 3x^4 + 4x^3 - x^2 - 2x + 2 &= (x^4 - 2x^3 + x^2 + x - 1)(x - 1) \\ &+ (x^3 - x^2 + 1) \\ x^4 - 2x^3 + x^2 + x - 1 &= (x^3 - x^2 + 1)(x - 1) \\ &+ 0 \end{aligned}$$

[解答]  $x^3 - x^2 + 1$

## A.9

[問題]  $x^5 + 2x^4 + x^3 - x - 1, \quad x^4 + 3x^3 + 3x^2 + 2x + 1$

[計算]

$$\begin{aligned} x^5 + 2x^4 + x^3 - x - 1 &= (x^4 + 3x^3 + 3x^2 + 2x + 1)(x - 1) + (x^3 + x^2) \\ x^4 + 3x^3 + 3x^2 + 2x + 1 &= (x^3 + x^2)(x + 2) + (x^2 + 2x + 1) \\ x^3 + x^2 &= (x^2 + 2x + 1)(x - 1) + (x + 1) \\ x^2 + 2x + 1 &= (x + 1)(x + 1) + 0 \end{aligned}$$

[解答]  $x + 1$

## A.10

[問題]  $x^4 + x^3 - x^2 - 4x - 2, \quad x^3 - 2x - 1$

[計算]

$$\begin{aligned} x^4 + x^3 - x^2 - 4x - 2 &= (x^3 - 2x - 1)(x + 1) + (x^2 - x - 1) \\ x^3 - 2x - 1 &= (x^2 - x - 1)(x + 1) + 0 \end{aligned}$$

[解答]  $x^2 - x - 1$

## A.11

[問題]  $x^6 - x^5 - x^4 + 8x^2 - 3x - 6, \quad x^5 - 2x^4 + 9x - 9$

[計算]

$$\begin{aligned} x^6 - x^5 - x^4 + 8x^2 - 3x - 6 &= (x^5 - 2x^4 + 9x - 9)(x + 1) \\ &\quad + (x^4 - x^2 - 3x + 3) \\ x^5 - 2x^4 + 9x - 9 &= (x^4 - x^2 - 3x + 3)(x - 2) \\ &\quad + (x^3 + x^2 - 3) \\ x^4 - x^2 - 3x + 3 &= (x^3 + x^2 - 3)(x - 1) \\ &\quad + 0 \end{aligned}$$

[解答]  $x^3 + x^2 - 3$

A.12

[問題]  $x^7 + x^6 - 2x^5 - x^4 + 3x^3 + 3x^2 - 4x + 3, \quad x^6 - 3x^4 + 3x^3 + 2x^2 - 3x + 2$

[計算] 
$$\begin{aligned} x^7 + x^6 - 2x^5 - x^4 + 3x^3 + 3x^2 - 4x + 3 &= (x^6 - 3x^4 + 3x^3 + 2x^2 - 3x + 2)(x + 1) \\ &\quad + (x^5 - x^4 - 2x^3 + 4x^2 - 3x + 1) \\ x^6 - 3x^4 + 3x^3 + 2x^2 - 3x + 2 &= (x^5 - x^4 - 2x^3 + 4x^2 - 3x + 1)(x + 1) \\ &\quad + (x^3 + x^2 - x + 1) \\ x^5 - x^4 - 2x^3 + 4x^2 - 3x + 1 &= (x^3 + x^2 - x + 1)(x^2 - 2x + 1) \\ &\quad + 0 \end{aligned}$$

[解答]  $x^3 + x^2 - x + 1$

A.13

[問題]  $x^5 - 4x^4 + 6x^3 - 7x + 4, \quad x^4 - 3x^3 + 2x^2 + 3x - 3$

[計算] 
$$\begin{aligned} x^5 - 4x^4 + 6x^3 - 7x + 4 &= (x^4 - 3x^3 + 2x^2 + 3x - 3)(x - 1) + (x^3 - x^2 - x + 1) \\ x^4 - 3x^3 + 2x^2 + 3x - 3 &= (x^3 - x^2 - x + 1)(x - 2) + (x^2 - 1) \\ x^3 - x^2 - x + 1 &= (x^2 - 1)(x - 1) + 0 \end{aligned}$$

[解答]  $x^2 - 1$

A.14

[問題]  $x^6 - 2x^5 + 4x^3 - 7x^2 + 4x - 3, \quad x^5 - x^4 - 2x^3 + 2x^2 - 2x + 1$

[計算] 
$$\begin{aligned} x^6 - 2x^5 + 4x^3 - 7x^2 + 4x - 3 &= (x^5 - x^4 - 2x^3 + 2x^2 - 2x + 1)(x - 1) \\ &\quad + (x^4 - 3x^2 + x - 2) \\ x^5 - x^4 - 2x^3 + 2x^2 - 2x + 1 &= (x^4 - 3x^2 + x - 2)(x - 1) \\ &\quad + (x^3 - 2x^2 + x - 1) \\ x^4 - 3x^2 + x - 2 &= (x^3 - 2x^2 + x - 1)(x + 2) \\ &\quad + 0 \end{aligned}$$

[解答]  $x^3 - 2x^2 + x - 1$

A.15

[問題]  $x^6 + 4x^5 + 3x^4 - 8x^3 - 9x^2 - 12x - 11, \quad x^5 + 2x^4 - 2x^3 - 3x^2 - 3x - 5$

$$x^6 + 4x^5 + 3x^4 - 8x^3 - 9x^2 - 12x - 11 = (x^5 + 2x^4 - 2x^3 - 3x^2 - 3x - 5)(x + 2)$$

$$+ (x^4 - x^3 - x - 1)$$

$$x^5 + 2x^4 - 2x^3 - 3x^2 - 3x - 5 = (x^4 - x^3 - x - 1)(x + 3)$$

$$+ (x^3 - 2x^2 + x - 2)$$

[計算]

$$x^4 - x^3 - x - 1 = (x^3 - 2x^2 + x - 2)(x + 1)$$

$$+ (x^2 + 1)$$

$$x^3 - 2x^2 + x - 2 = (x^2 + 1)(x - 2)$$

$$+ 0$$

[解答]  $x^2 + 1$

A.16

[問題]  $x^6 + 3x^5 + 5x^4 + 7x^3 + 6x^2 + 4x + 2, \quad x^5 + 2x^4 + 3x^3 + 3x^2 + 2x + 1$

$$x^6 + 3x^5 + 5x^4 + 7x^3 + 6x^2 + 4x + 2 = (x^5 + 2x^4 + 3x^3 + 3x^2 + 2x + 1)(x + 1)$$

$$+ (x^3 + x^2 + x + 1)$$

[計算]

$$x^5 + 2x^4 + 3x^3 + 3x^2 + 2x + 1 = (x^3 + x^2 + x + 1)(x^2 + x + 1)$$

$$+ 0$$

[解答]  $x^3 + x^2 + x + 1$

A.17

[問題]  $x^4 + 3x^3 + 4x^2 + 7x + 6, \quad x^3 + 2x^2 + 2x + 4$

[計算]  $x^4 + 3x^3 + 4x^2 + 7x + 6 = (x^3 + 2x^2 + 2x + 4)(x + 1) + (x + 2)$

$$x^3 + 2x^2 + 2x + 4 = (x + 2)(x^2 + 2) + 0$$

[解答]  $x + 2$

A.18

[問題]  $x^7 + x^6 + x^5 + x^4 + 2x^3 - x - 1, \quad x^6 + x^2 - x$

$$\begin{aligned} x^7 + x^6 + x^5 + x^4 + 2x^3 - x - 1 &= (x^6 + x^2 - x)(x + 1) \\ &+ (x^5 + x^4 + x^3 - 1) \\ \text{[計算]} \quad x^6 + x^2 - x &= (x^5 + x^4 + x^3 - 1)(x - 1) \\ &+ (x^3 + x^2 - 1) \\ x^5 + x^4 + x^3 - 1 &= (x^3 + x^2 - 1)(x^2 + 1) \\ &+ 0 \end{aligned}$$

[解答]  $x^3 + x^2 - 1$

A.19

[問題]  $x^4 + x, \quad x^3 + x^2 + x + 1$

$$\begin{aligned} x^4 + x &= (x^3 + x^2 + x + 1)(x - 1) + (x + 1) \\ \text{[計算]} \quad x^3 + x^2 + x + 1 &= (x + 1)(x^2 + 1) + 0 \end{aligned}$$

[解答]  $x + 1$

A.20

[問題]  $x^5 - 2x^4 - 3x^3 + 8x^2 - 6, \quad x^4 - x^3 - 4x^2 + 3x + 3$

$$\begin{aligned} x^5 - 2x^4 - 3x^3 + 8x^2 - 6 &= (x^4 - x^3 - 4x^2 + 3x + 3)(x - 1) + (x^2 - 3) \\ \text{[計算]} \quad x^4 - x^3 - 4x^2 + 3x + 3 &= (x^2 - 3)(x^2 - x - 1) + 0 \end{aligned}$$

[解答]  $x^2 - 3$

A.21

[問題]  $x^4 - 2x^2 + 1, \quad x^2 - 1$

[計算]  $x^4 - 2x^2 + 1 = (x^2 - 1)(x^2 - 1) + 0$

[解答]  $x^2 - 1$