

代幾 I 計算演習 [問題] (2006/11/30)

問. 次の行列の行列式を求めなさい

Q.1

$$\begin{vmatrix} 7 & 6 & 4 & 0 \\ 4 & -1 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ -6 & 3 & -7 & 3 \end{vmatrix}$$

Q.2

$$\begin{vmatrix} -6 & -7 & -8 & 1 \\ 4 & 0 & 0 & 0 \\ -1 & 4 & -6 & 0 \\ 1 & 0 & 1 & 0 \end{vmatrix}$$

Q.3

$$\begin{vmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ -5 & 0 & 0 & 1 & 0 & 0 \\ -3 & 1 & -3 & 8 & 3 & 1 \\ -6 & 0 & -9 & 7 & 3 & 0 \\ -9 & 0 & 2 & 6 & 0 & 0 \\ -8 & 0 & -2 & -6 & -6 & 4 \end{vmatrix}$$

Q.4

$$\begin{vmatrix} 0 & 0 & 0 & 4 & 0 & 0 \\ -3 & 5 & 6 & 1 & 5 & 2 \\ 2 & 0 & 0 & 3 & 9 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ -2 & 8 & 3 & 9 & -7 & 0 \\ 7 & 1 & 0 & 7 & -4 & 0 \end{vmatrix}$$

Q.5

$$\begin{vmatrix} 3 & 0 & -5 & 0 & 3 & 0 \\ -1 & 3 & -6 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 3 & 9 & 7 & 0 & 4 & 2 \\ 5 & -9 & -6 & 4 & 6 & 8 \end{vmatrix}$$

Q.6

$$\begin{vmatrix} 0 & 6 & 0 & 0 & 8 & 3 \\ -3 & 1 & -9 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & -5 & 0 \\ 0 & 8 & 3 & 0 & -2 & 4 \\ 4 & -8 & 1 & 0 & 6 & -4 \end{vmatrix}$$

Q.7

$$\begin{vmatrix} 4 & 8 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 9 & 2 & 1 & 0 \\ -7 & -6 & 2 & 1 \end{vmatrix}$$

Q.8

$$\begin{vmatrix} 6 & -7 & 0 & 3 & 0 & -6 \\ -2 & 0 & 0 & 0 & 4 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 8 & -5 & 2 & 4 & 9 & -1 \\ -7 & 1 & 0 & 0 & 5 & 0 \\ 6 & -9 & 0 & 0 & 1 & 3 \end{vmatrix}$$

Q.9

$$\begin{vmatrix} 6 & -5 & 3 & 8 & -3 \\ 1 & 6 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 & 0 \\ 4 & -7 & 0 & 0 & 0 \\ 0 & 6 & 0 & 1 & -7 \end{vmatrix}$$

Q.10

$$\begin{vmatrix} 0 & 0 & 0 & 3 & 7 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ -7 & 1 & -5 & -9 & 2 & 0 \\ 3 & 0 & 8 & 0 & -4 & 0 \\ 0 & 0 & 4 & 1 & 4 & 0 \\ -1 & -9 & -9 & -2 & -8 & 1 \end{vmatrix}$$

代幾 I 計算演習 [解答] (2006/11/30)

A.1

$$\begin{aligned}
 \begin{vmatrix} 7 & 6 & 4 & 0 \\ 4 & -1 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ -6 & 3 & -7 & 3 \end{vmatrix} &= \begin{vmatrix} 4 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 6 & 7 & 4 & 0 \\ 3 & -6 & -7 & 3 \end{vmatrix} && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 &= 4 \begin{vmatrix} 4 & 0 & 0 \\ 7 & 4 & 0 \\ -6 & -7 & 3 \end{vmatrix} \\
 &= 16 \begin{vmatrix} 4 & 0 \\ -7 & 3 \end{vmatrix} \\
 &= 64 \begin{vmatrix} 3 \end{vmatrix} \\
 &= 192
 \end{aligned}$$

A.2

$$\begin{aligned}
 \begin{vmatrix} -6 & -7 & -8 & 1 \\ 4 & 0 & 0 & 0 \\ -1 & 4 & -6 & 0 \\ 1 & 0 & 1 & 0 \end{vmatrix} &= - \begin{vmatrix} 4 & 0 & 0 & 0 \\ -6 & -7 & -8 & 1 \\ -1 & 4 & -6 & 0 \\ 1 & 0 & 1 & 0 \end{vmatrix} && \text{1 行目と 2 行目を交換} \\
 &= -4 \begin{vmatrix} -7 & -8 & 1 \\ 4 & -6 & 0 \\ 0 & 1 & 0 \end{vmatrix} \\
 &= -4 \begin{vmatrix} 1 & 0 & 0 \\ -6 & 4 & 0 \\ -8 & -7 & 1 \end{vmatrix} && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 &= -4 \begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix} \\
 &= -16 \begin{vmatrix} 1 \end{vmatrix} \\
 &= -16
 \end{aligned}$$

A.3

$$\begin{aligned}
 & \begin{vmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ -5 & 0 & 0 & 1 & 0 & 0 \\ -3 & 1 & -3 & 8 & 3 & 1 \\ -6 & 0 & -9 & 7 & 3 & 0 \\ -9 & 0 & 2 & 6 & 0 & 0 \\ -8 & 0 & -2 & -6 & -6 & 4 \end{vmatrix} = 2 \begin{vmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & -3 & 8 & 3 & 1 \\ 0 & -9 & 7 & 3 & 0 \\ 0 & 2 & 6 & 0 & 0 \\ 0 & -2 & -6 & -6 & 4 \end{vmatrix} \\
 & = -2 \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 8 & -3 & 1 & 3 & 1 \\ 7 & -9 & 0 & 3 & 0 \\ 6 & 2 & 0 & 0 & 0 \\ -6 & -2 & 0 & -6 & 4 \end{vmatrix} \quad \text{1 列目と 3 列目を交換} \\
 & = -2 \begin{vmatrix} -3 & 1 & 3 & 1 \\ -9 & 0 & 3 & 0 \\ 2 & 0 & 0 & 0 \\ -2 & 0 & -6 & 4 \end{vmatrix} \\
 & = 2 \begin{vmatrix} 2 & 0 & 0 & 0 \\ -9 & 0 & 3 & 0 \\ -3 & 1 & 3 & 1 \\ -2 & 0 & -6 & 4 \end{vmatrix} \quad \text{1 行目と 3 行目を交換} \\
 & = 4 \begin{vmatrix} 0 & 3 & 0 \\ 1 & 3 & 1 \\ 0 & -6 & 4 \end{vmatrix} \\
 & = -4 \begin{vmatrix} 3 & 0 & 0 \\ 3 & 1 & 1 \\ -6 & 0 & 4 \end{vmatrix} \quad \text{1 列目と 2 列目を交換} \\
 & = -12 \begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} \\
 & = -12 \begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} \quad \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 & = -48 \begin{vmatrix} 1 \end{vmatrix} \\
 & = -48
 \end{aligned}$$

A.4

$$\begin{aligned}
 & \begin{vmatrix} 0 & 0 & 0 & 4 & 0 & 0 \\ -3 & 5 & 6 & 1 & 5 & 2 \\ 2 & 0 & 0 & 3 & 9 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ -2 & 8 & 3 & 9 & -7 & 0 \\ 7 & 1 & 0 & 7 & -4 & 0 \end{vmatrix} = - \begin{vmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 1 & 5 & 6 & -3 & 5 & 2 \\ 3 & 0 & 0 & 2 & 9 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 9 & 8 & 3 & -2 & -7 & 0 \\ 7 & 1 & 0 & 7 & -4 & 0 \end{vmatrix} & \text{1 列目と 4 列目を交換} \\
 & = -4 \begin{vmatrix} 5 & 6 & -3 & 5 & 2 \\ 0 & 0 & 2 & 9 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 8 & 3 & -2 & -7 & 0 \\ 1 & 0 & 7 & -4 & 0 \end{vmatrix} \\
 & = -4 \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 9 & 0 & 2 & 0 & 0 \\ 5 & 6 & -3 & 5 & 2 \\ -7 & 3 & -2 & 8 & 0 \\ -4 & 0 & 7 & 1 & 0 \end{vmatrix} & \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 4 列目を交換} \end{array} \\
 & = -8 \begin{vmatrix} 0 & 2 & 0 & 0 \\ 6 & -3 & 5 & 2 \\ 3 & -2 & 8 & 0 \\ 0 & 7 & 1 & 0 \end{vmatrix} \\
 & = 8 \begin{vmatrix} 2 & 0 & 0 & 0 \\ -3 & 6 & 5 & 2 \\ -2 & 3 & 8 & 0 \\ 7 & 0 & 1 & 0 \end{vmatrix} & \text{1 列目と 2 列目を交換} \\
 & = 16 \begin{vmatrix} 6 & 5 & 2 \\ 3 & 8 & 0 \\ 0 & 1 & 0 \end{vmatrix} \\
 & = 16 \begin{vmatrix} 1 & 0 & 0 \\ 8 & 3 & 0 \\ 5 & 6 & 2 \end{vmatrix} & \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 & = 16 \begin{vmatrix} 3 & 0 \\ 6 & 2 \end{vmatrix} \\
 & = 48 \begin{vmatrix} 2 \end{vmatrix} \\
 & = 96
 \end{aligned}$$

A.5

$$\begin{aligned}
 & \begin{vmatrix} 3 & 0 & -5 & 0 & 3 & 0 \\ -1 & 3 & -6 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 3 & 9 & 7 & 0 & 4 & 2 \\ 5 & -9 & -6 & 4 & 6 & 8 \end{vmatrix} = \begin{vmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 3 & -6 & 0 & -1 & 0 \\ -4 & 0 & 4 & 0 & 0 & 0 \\ 3 & 0 & -5 & 0 & 3 & 0 \\ 4 & 9 & 7 & 0 & 3 & 2 \\ 6 & -9 & -6 & 4 & 5 & 8 \end{vmatrix} & \begin{array}{l} \text{1 行目と 4 行目を交換} \\ \text{1 列目と 5 列目を交換} \end{array} \\
 & = 3 \begin{vmatrix} 3 & -6 & 0 & -1 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & -5 & 0 & 3 & 0 \\ 9 & 7 & 0 & 3 & 2 \\ -9 & -6 & 4 & 5 & 8 \end{vmatrix} \\
 & = 3 \begin{vmatrix} 4 & 0 & 0 & 0 & 0 \\ -6 & 3 & 0 & -1 & 0 \\ -5 & 0 & 0 & 3 & 0 \\ 7 & 9 & 0 & 3 & 2 \\ -6 & -9 & 4 & 5 & 8 \end{vmatrix} & \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 & = 12 \begin{vmatrix} 3 & 0 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 9 & 0 & 3 & 2 \\ -9 & 4 & 5 & 8 \end{vmatrix} \\
 & = 12 \begin{vmatrix} 3 & 0 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 3 & 0 & 9 & 2 \\ 5 & 4 & -9 & 8 \end{vmatrix} & \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
 & = 36 \begin{vmatrix} 0 & 3 & 0 \\ 0 & 9 & 2 \\ 4 & -9 & 8 \end{vmatrix} \\
 & = -36 \begin{vmatrix} 3 & 0 & 0 \\ 9 & 0 & 2 \\ -9 & 4 & 8 \end{vmatrix} & \text{1 列目と 2 列目を交換} \\
 & = -108 \begin{vmatrix} 0 & 2 \\ 4 & 8 \end{vmatrix} \\
 & = 108 \begin{vmatrix} 2 & 0 \\ 8 & 4 \end{vmatrix} & \text{1 列目と 2 列目を交換} \\
 & = 216 \begin{vmatrix} 4 \end{vmatrix} \\
 & = 864
 \end{aligned}$$

A.6

$$\begin{aligned}
 & \begin{vmatrix} 0 & 6 & 0 & 0 & 8 & 3 \\ -3 & 1 & -9 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & -5 & 0 \\ 0 & 8 & 3 & 0 & -2 & 4 \\ 4 & -8 & 1 & 0 & 6 & -4 \end{vmatrix} = \begin{vmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & -9 & 2 & -3 & 5 \\ 8 & 6 & 0 & 0 & 0 & 3 \\ -5 & 4 & 0 & 0 & 0 & 0 \\ -2 & 8 & 3 & 0 & 0 & 4 \\ 6 & -8 & 1 & 0 & 4 & -4 \end{vmatrix} \quad \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 5 列目を交換} \end{array} \\
 & = 3 \begin{vmatrix} 1 & -9 & 2 & -3 & 5 \\ 6 & 0 & 0 & 0 & 3 \\ 4 & 0 & 0 & 0 & 0 \\ 8 & 3 & 0 & 0 & 4 \\ -8 & 1 & 0 & 4 & -4 \end{vmatrix} \\
 & = -3 \begin{vmatrix} 4 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 3 \\ 1 & -9 & 2 & -3 & 5 \\ 8 & 3 & 0 & 0 & 4 \\ -8 & 1 & 0 & 4 & -4 \end{vmatrix} \quad \text{1 行目と 3 行目を交換} \\
 & = -12 \begin{vmatrix} 0 & 0 & 0 & 3 \\ -9 & 2 & -3 & 5 \\ 3 & 0 & 0 & 4 \\ 1 & 0 & 4 & -4 \end{vmatrix} \\
 & = 12 \begin{vmatrix} 3 & 0 & 0 & 0 \\ 5 & 2 & -3 & -9 \\ 4 & 0 & 0 & 3 \\ -4 & 0 & 4 & 1 \end{vmatrix} \quad \text{1 列目と 4 列目を交換} \\
 & = 36 \begin{vmatrix} 2 & -3 & -9 \\ 0 & 0 & 3 \\ 0 & 4 & 1 \end{vmatrix} \\
 & = 36 \begin{vmatrix} 3 & 0 & 0 \\ -9 & -3 & 2 \\ 1 & 4 & 0 \end{vmatrix} \quad \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
 & = 108 \begin{vmatrix} -3 & 2 \\ 4 & 0 \end{vmatrix} \\
 & = -108 \begin{vmatrix} 4 & 0 \\ -3 & 2 \end{vmatrix} \quad \text{1 行目と 2 行目を交換} \\
 & = -432 \begin{vmatrix} 2 \end{vmatrix} \\
 & = -864
 \end{aligned}$$

A.7

$$\begin{aligned} \begin{vmatrix} 4 & 8 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 9 & 2 & 1 & 0 \\ -7 & -6 & 2 & 1 \end{vmatrix} &= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 8 & 4 & 0 & 0 \\ 2 & 9 & 1 & 0 \\ -6 & -7 & 2 & 1 \end{vmatrix} && \begin{array}{l} \text{1行目と2行目を交換} \\ \text{1列目と2列目を交換} \end{array} \\ &= \begin{vmatrix} 4 & 0 & 0 \\ 9 & 1 & 0 \\ -7 & 2 & 1 \end{vmatrix} \\ &= 4 \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \\ &= 4 \begin{vmatrix} 1 \\ 1 \end{vmatrix} \\ &= 4 \end{aligned}$$

A.8

$$\begin{aligned}
 & \begin{vmatrix} 6 & -7 & 0 & 3 & 0 & -6 \\ -2 & 0 & 0 & 0 & 4 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 8 & -5 & 2 & 4 & 9 & -1 \\ -7 & 1 & 0 & 0 & 5 & 0 \\ 6 & -9 & 0 & 0 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 4 & 0 \\ 6 & -7 & 0 & 3 & 0 & -6 \\ 8 & -5 & 2 & 4 & 9 & -1 \\ -7 & 1 & 0 & 0 & 5 & 0 \\ 6 & -9 & 0 & 0 & 1 & 3 \end{vmatrix} & \text{1 行目と 3 行目を交換} \\
 & = -4 \begin{vmatrix} 0 & 0 & 0 & 4 & 0 \\ -7 & 0 & 3 & 0 & -6 \\ -5 & 2 & 4 & 9 & -1 \\ 1 & 0 & 0 & 5 & 0 \\ -9 & 0 & 0 & 1 & 3 \end{vmatrix} \\
 & = 4 \begin{vmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -7 & -6 \\ 9 & 2 & 4 & -5 & -1 \\ 5 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -9 & 3 \end{vmatrix} & \text{1 列目と 4 列目を交換} \\
 & = 16 \begin{vmatrix} 0 & 3 & -7 & -6 \\ 2 & 4 & -5 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -9 & 3 \end{vmatrix} \\
 & = 16 \begin{vmatrix} 1 & 0 & 0 & 0 \\ -5 & 4 & 2 & -1 \\ -7 & 3 & 0 & -6 \\ -9 & 0 & 0 & 3 \end{vmatrix} & \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
 & = 16 \begin{vmatrix} 4 & 2 & -1 \\ 3 & 0 & -6 \\ 0 & 0 & 3 \end{vmatrix} \\
 & = 16 \begin{vmatrix} 3 & 0 & 0 \\ -6 & 0 & 3 \\ -1 & 2 & 4 \end{vmatrix} & \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
 & = 48 \begin{vmatrix} 0 & 3 \\ 2 & 4 \end{vmatrix} \\
 & = -48 \begin{vmatrix} 3 & 0 \\ 4 & 2 \end{vmatrix} & \text{1 列目と 2 列目を交換} \\
 & = -144 \begin{vmatrix} 2 \end{vmatrix} \\
 & = -288
 \end{aligned}$$

A.9

$$\begin{aligned} \begin{vmatrix} 6 & -5 & 3 & 8 & -3 \\ 1 & 6 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 & 0 \\ 4 & -7 & 0 & 0 & 0 \\ 0 & 6 & 0 & 1 & -7 \end{vmatrix} &= \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 6 & 1 & 0 & 0 & 3 \\ -5 & 6 & 3 & 8 & -3 \\ -7 & 4 & 0 & 0 & 0 \\ 6 & 0 & 0 & 1 & -7 \end{vmatrix} && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\ &= 2 \begin{vmatrix} 1 & 0 & 0 & 3 \\ 6 & 3 & 8 & -3 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & -7 \end{vmatrix} \\ &= -2 \begin{vmatrix} 4 & 0 & 0 & 0 \\ 6 & 3 & 8 & -3 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{vmatrix} && \text{1 行目と 3 行目を交換} \\ &= -8 \begin{vmatrix} 3 & 8 & -3 \\ 0 & 0 & 3 \\ 0 & 1 & -7 \end{vmatrix} \\ &= -8 \begin{vmatrix} 3 & 0 & 0 \\ -3 & 8 & 3 \\ -7 & 1 & 0 \end{vmatrix} && \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\ &= -24 \begin{vmatrix} 8 & 3 \\ 1 & 0 \end{vmatrix} \\ &= 24 \begin{vmatrix} 1 & 0 \\ 8 & 3 \end{vmatrix} && \text{1 行目と 2 行目を交換} \\ &= 24 \begin{vmatrix} 3 \end{vmatrix} \\ &= 72 \end{aligned}$$

A.10

$$\begin{aligned}
 & \begin{vmatrix} 0 & 0 & 0 & 3 & 7 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ -7 & 1 & -5 & -9 & 2 & 0 \\ 3 & 0 & 8 & 0 & -4 & 0 \\ 0 & 0 & 4 & 1 & 4 & 0 \\ -1 & -9 & -9 & -2 & -8 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 3 & 0 & 0 \\ 2 & 1 & -5 & -9 & -7 & 0 \\ -4 & 0 & 8 & 0 & 3 & 0 \\ 4 & 0 & 4 & 1 & 0 & 0 \\ -8 & -9 & -9 & -2 & -1 & 1 \end{vmatrix} & \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 5 列目を交換} \end{array} \\
 & = 4 \begin{vmatrix} 0 & 0 & 3 & 0 & 0 \\ 1 & -5 & -9 & -7 & 0 \\ 0 & 8 & 0 & 3 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ -9 & -9 & -2 & -1 & 1 \end{vmatrix} \\
 & = -4 \begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ -9 & -5 & 1 & -7 & 0 \\ 0 & 8 & 0 & 3 & 0 \\ 1 & 4 & 0 & 0 & 0 \\ -2 & -9 & -9 & -1 & 1 \end{vmatrix} & \text{1 列目と 3 列目を交換} \\
 & = -12 \begin{vmatrix} -5 & 1 & -7 & 0 \\ 8 & 0 & 3 & 0 \\ 4 & 0 & 0 & 0 \\ -9 & -9 & -1 & 1 \end{vmatrix} \\
 & = 12 \begin{vmatrix} 4 & 0 & 0 & 0 \\ 8 & 0 & 3 & 0 \\ -5 & 1 & -7 & 0 \\ -9 & -9 & -1 & 1 \end{vmatrix} & \text{1 行目と 3 行目を交換} \\
 & = 48 \begin{vmatrix} 0 & 3 & 0 \\ 1 & -7 & 0 \\ -9 & -1 & 1 \end{vmatrix} \\
 & = -48 \begin{vmatrix} 3 & 0 & 0 \\ -7 & 1 & 0 \\ -1 & -9 & 1 \end{vmatrix} & \text{1 列目と 2 列目を交換} \\
 & = -144 \begin{vmatrix} 1 & 0 \\ -9 & 1 \end{vmatrix} \\
 & = -144 \begin{vmatrix} 1 \end{vmatrix} \\
 & = -144
 \end{aligned}$$