

代幾 I 計算演習 [問題] (2006/12/07)

問. 次の行列の行列式を求めなさい

Q.1

$$\begin{vmatrix} -1 & 2 & -1 & -2 \\ 1 & -2 & 1 & -1 \\ -2 & 2 & 1 & 0 \\ -2 & 1 & 1 & 2 \end{vmatrix}$$

Q.6

$$\begin{vmatrix} 2 & -1 & -1 & 1 \\ -1 & -1 & -2 & -1 \\ 1 & -1 & 1 & 1 \\ 0 & -1 & 1 & -1 \end{vmatrix}$$

Q.2

$$\begin{vmatrix} 1 & -1 & -1 & 1 \\ 2 & 0 & -2 & -2 \\ 1 & -1 & -1 & -1 \\ 0 & 0 & -2 & 2 \end{vmatrix}$$

Q.7

$$\begin{vmatrix} -1 & 1 & 0 & -1 \\ -2 & 0 & 2 & 2 \\ 2 & -2 & 2 & -1 \\ -2 & 2 & -1 & 2 \end{vmatrix}$$

Q.3

$$\begin{vmatrix} -1 & -1 & -2 & -1 \\ 0 & 2 & -2 & 0 \\ -1 & -2 & -2 & -2 \\ 0 & 1 & -2 & 1 \end{vmatrix}$$

Q.8

$$\begin{vmatrix} -1 & 0 & -1 & 0 \\ -2 & 0 & -2 & 0 \\ -1 & 2 & -1 & 0 \\ -2 & 0 & 0 & -1 \end{vmatrix}$$

Q.4

$$\begin{vmatrix} 2 & 1 & 1 & 0 \\ -2 & 0 & -1 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & -1 & -1 & -2 \end{vmatrix}$$

Q.9

$$\begin{vmatrix} -2 & 0 & -1 & 1 \\ 0 & 0 & -2 & 2 \\ -1 & 0 & -1 & -2 \\ 2 & 1 & 2 & 2 \end{vmatrix}$$

Q.5

$$\begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & -1 & 1 & -1 \\ -1 & 0 & -2 & -1 \\ -2 & 2 & 1 & 2 \end{vmatrix}$$

Q.10

$$\begin{vmatrix} -1 & -2 & -1 & 1 \\ 1 & -2 & -2 & 0 \\ 1 & -2 & 1 & -1 \\ 1 & 1 & 2 & 2 \end{vmatrix}$$

代幾 I 計算演習 [解答] (2006/12/07)

A.1

A.2

$$\begin{aligned}
 \begin{vmatrix} -1 & 2 & -1 & -2 \\ 1 & -2 & 1 & -1 \\ -2 & 2 & 1 & 0 \\ -2 & 1 & 1 & 2 \end{vmatrix} &= 1 \begin{vmatrix} -1 & 2 & -1 & -2 \\ 0 & 0 & 0 & -3 \\ -2 & 2 & 1 & 0 \\ -2 & 1 & 1 & 2 \end{vmatrix} \\
 &= 1 \begin{vmatrix} -1 & 2 & -1 & -2 \\ 0 & 0 & 0 & -3 \\ 0 & -2 & 3 & 4 \\ -2 & 1 & 1 & 2 \end{vmatrix} \\
 &= 1 \begin{vmatrix} -1 & 2 & -1 & -2 \\ 0 & 0 & 0 & -3 \\ 0 & -2 & 3 & 4 \\ 0 & -3 & 3 & 6 \end{vmatrix} \\
 &= -1 \begin{vmatrix} 0 & 0 & -3 \\ -2 & 3 & 4 \\ -3 & 3 & 6 \end{vmatrix} \\
 &= 1 \begin{vmatrix} -2 & 3 & 4 \\ 0 & 0 & -3 \\ -3 & 3 & 6 \end{vmatrix} \\
 &= 1 \begin{vmatrix} -2 & 3 & 4 \\ 0 & 0 & -3 \\ 0 & -\frac{3}{2} & 0 \end{vmatrix} \\
 &= -2 \begin{vmatrix} 0 & -3 \\ -\frac{3}{2} & 0 \end{vmatrix} \\
 &= -2 \times (0 \times 0 - (-3) \times (-\frac{3}{2})) \\
 &= 9
 \end{aligned}$$

左 R(2,1;1) ; 2行目に1行目を1倍して、加える

左 R(3,1;-2) ; 3行目に1行目を-2倍して、加える

左 R(4,1;-2) ; 4行目に1行目を-2倍して、加える

$$\begin{vmatrix} 1 & -1 & -1 & 1 \\ 2 & 0 & -2 & -2 \\ 1 & -1 & -1 & -1 \\ 0 & 0 & -2 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & -1 & -1 & 1 \\ 0 & 2 & 0 & -4 \\ 1 & -1 & -1 & -1 \\ 0 & 0 & -2 & 2 \end{vmatrix}$$

左 P(3,2) ; 3行目と2行目を交換

左 R(4,2;-3/2) ; 4行目に2行目を-3/2倍して、加える

$$\begin{vmatrix} 1 & -1 & -1 & 1 \\ 0 & 2 & 0 & -4 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & 2 \end{vmatrix}$$

$$\begin{aligned}
 &= 2 \begin{vmatrix} 2 & 0 & -4 \\ 0 & -2 & 2 \\ 0 & -2 & 2 \end{vmatrix} \\
 &= 2 \times (0 \times 2 - (-2) \times (-2)) \\
 &= -8
 \end{aligned}$$

A.3

$$\begin{aligned}
 & \begin{vmatrix} -1 & -1 & -2 & -1 \\ 0 & 2 & -2 & 0 \\ -1 & -2 & -2 & -2 \\ 0 & 1 & -2 & 1 \end{vmatrix} = 1 \begin{vmatrix} -1 & -1 & -2 & -1 \\ 0 & 2 & -2 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & -2 & 1 \end{vmatrix} \\
 & = -1 \begin{vmatrix} 2 & -2 & 0 \\ -1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix} \\
 & = -1 \begin{vmatrix} 2 & -2 & 0 \\ 0 & -1 & -1 \\ 1 & -2 & 1 \end{vmatrix} \\
 & = -1 \begin{vmatrix} 2 & -2 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & 1 \end{vmatrix} \\
 & = -2 \begin{vmatrix} -1 & -1 \\ -1 & 1 \end{vmatrix} \\
 & = -2 \times ((-1) \times 1 - (-1) \times (-1)) \\
 & = 4
 \end{aligned}$$

A.4

$$\begin{aligned}
 & \begin{vmatrix} 2 & 1 & 1 & 0 \\ -2 & 0 & -1 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & -1 & -1 & -2 \end{vmatrix} = 1 \begin{vmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & -1 & -1 & -2 \end{vmatrix} \quad \text{左} \\
 & = 1 \begin{vmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & \frac{1}{2} & \frac{3}{2} & 2 \\ 1 & -1 & -1 & -2 \end{vmatrix} \quad \text{左} \\
 & \begin{vmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & \frac{1}{2} & \frac{3}{2} & 2 \\ 0 & -\frac{3}{2} & -\frac{3}{2} & -2 \end{vmatrix} \quad \text{左 R(3,1;-1); 3行目に1行目を}-1\text{倍して、加える} \\
 & = 2 \begin{vmatrix} 1 & 0 & 2 \\ \frac{1}{2} & \frac{3}{2} & 2 \\ -\frac{3}{2} & -\frac{3}{2} & -2 \\ 1 & 0 & 2 \end{vmatrix} \\
 & \begin{vmatrix} 1 & 0 & 2 \\ \frac{1}{2} & \frac{3}{2} & 2 \\ -\frac{3}{2} & -\frac{3}{2} & -2 \\ -\frac{3}{2} & -\frac{3}{2} & -2 \end{vmatrix} \quad \text{左 R(3,2;\frac{1}{2}); 3行目に2行目を}\frac{1}{2}\text{倍して、加える} \\
 & \begin{vmatrix} 1 & 0 & 2 \\ 0 & -\frac{3}{2} & 1 \\ 0 & -\frac{3}{2} & 1 \end{vmatrix} \quad \text{左 R(4,2;-\frac{1}{2}); 4行目に3行目を}1\frac{1}{2}\text{倍して、加える} \\
 & = 2 \begin{vmatrix} \frac{3}{2} & 1 \\ -\frac{3}{2} & 1 \end{vmatrix} \\
 & = 2 \times (\frac{3}{2} \times 1 - 1 \times (-\frac{3}{2})) \\
 & = 6
 \end{aligned}$$

$$\begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & -1 & 1 & -1 \\ -1 & 0 & -2 & -1 \\ -2 & 2 & 1 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ -2 & 2 & 1 & 2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 4 & 1 & 6 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -1 & 1 & -1 \\ 1 & -2 & 1 \\ 4 & 1 & 6 \\ -1 & 1 & -1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -1 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 5 & 2 \end{vmatrix}$$

$$= -1 \begin{vmatrix} -1 & 0 \\ 5 & 2 \end{vmatrix}$$

$$= -1 \times ((-1) \times 2 - 0 \times 5)$$

$$= 2$$

$$\begin{vmatrix} 2 & -1 & -1 & 1 \\ 0 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & -1 & 1 & -1 \end{vmatrix} \xrightarrow{\text{左 R(3,1;1); 3行目に1行目を1倍して、加える}}$$

$$\begin{vmatrix} 2 & -1 & -1 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & -1 & 1 & -1 \end{vmatrix} \xrightarrow{\text{左 R(4,1;2); 4行目に1行目を2倍して、加える}}$$

$$= 2 \begin{vmatrix} -\frac{3}{2} & -\frac{5}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} \\ -1 & 1 & -1 \\ -\frac{3}{2} & -\frac{5}{2} & -\frac{1}{2} \end{vmatrix} \xrightarrow{\text{左 R(3,2;1); 3行目に2行目を1倍して、加える}}$$

$$\begin{vmatrix} -\frac{3}{2} & -\frac{5}{2} & -\frac{1}{2} \\ 0 & \frac{8}{3} & -\frac{2}{3} \\ 0 & \frac{8}{3} & -\frac{2}{3} \end{vmatrix} \xrightarrow{\text{左 R(4,2;4); 4行目に2行目を4倍して、加える}}$$

$$= -3 \begin{vmatrix} \frac{7}{3} & \frac{2}{3} \\ \frac{8}{3} & -\frac{2}{3} \end{vmatrix}$$

$$= -3 \times \left(\frac{7}{3} \times \left(-\frac{2}{3} \right) - \frac{2}{3} \times \frac{8}{3} \right)$$

$$= 10$$

A.7

$$\begin{aligned}
& \begin{vmatrix} -1 & 1 & 0 & -1 \\ -2 & 0 & 2 & 2 \\ 2 & -2 & 2 & -1 \\ -2 & 2 & -1 & 2 \end{vmatrix} = 1 \begin{vmatrix} -1 & 1 & 0 & -1 \\ 0 & -2 & 2 & 4 \\ 2 & -2 & 2 & -1 \\ -2 & 2 & -1 & 2 \end{vmatrix} \\
& = 1 \begin{vmatrix} -1 & 1 & 0 & -1 \\ 0 & -2 & 2 & 4 \\ 0 & 0 & 2 & -3 \\ -2 & 2 & -1 & 2 \end{vmatrix} \\
& = 1 \begin{vmatrix} -1 & 1 & 0 & -1 \\ 0 & -2 & 2 & 4 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & -1 & 4 \end{vmatrix} \\
& = -1 \begin{vmatrix} -2 & 2 & 4 \\ 0 & 2 & -3 \\ 0 & -1 & 4 \end{vmatrix} \\
& = 2 \begin{vmatrix} 2 & -3 \\ -1 & 4 \end{vmatrix} \\
& = 2 \times (2 \times 4 - (-3) \times (-1)) \\
& = 10
\end{aligned}$$

A.8

$$\begin{aligned}
& \begin{vmatrix} -1 & 0 & -1 & 0 \\ -2 & 0 & -2 & 0 \\ -1 & 2 & -1 & 0 \\ -2 & 0 & 0 & -1 \end{vmatrix} = 1 \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ -2 & 0 & 0 & -1 \end{vmatrix} \quad \text{左 R(1,1)} \\
& \text{左 R(2,1;-2); 2行目に1行目を} \frac{-1}{-2} \text{倍して、加える} \\
& = 1 \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -2 & 0 & 0 & -1 \end{vmatrix} \quad \text{左 R(3,1;2)} \\
& \text{左 R(3,1;2); 3行目に1行目を2倍して、加える} \\
& = 1 \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \end{vmatrix} \quad \text{左 R(4,1;-2)} \\
& \text{左 R(4,1;-2); 4行目に1行目を} \frac{-1}{-2} \text{倍して、加える} \\
& = 1 \begin{vmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 2 & -1 \end{vmatrix} \quad \text{左 P(1,2)} \\
& = 1 \begin{vmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & -1 \end{vmatrix} \\
& = 2 \begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} \\
& = 2 \times (0 \times (-1) - 0 \times 2) \\
& = 0
\end{aligned}$$

A.9

$$\begin{aligned}
 & \begin{vmatrix} -2 & 0 & -1 & 1 \\ 0 & 0 & -2 & 2 \\ -1 & 0 & -1 & -2 \\ 2 & 1 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} -2 & 0 & -1 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & -\frac{1}{2} & -\frac{5}{2} \\ 2 & 1 & 2 & 2 \end{vmatrix} \\
 & = 1 \begin{vmatrix} -2 & 0 & -1 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 1 & 1 & 3 \end{vmatrix} \\
 & = -2 \begin{vmatrix} 0 & -2 & 2 \\ 0 & -\frac{1}{2} & -\frac{5}{2} \\ 1 & 1 & 3 \end{vmatrix} \\
 & = 2 \begin{vmatrix} 1 & 1 & 3 \\ 0 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & -2 & 2 \end{vmatrix} \\
 & = 2 \begin{vmatrix} -\frac{1}{2} & -\frac{5}{2} \\ -2 & 2 \end{vmatrix} \\
 & = 2 \times ((-\frac{1}{2}) \times 2 - (-\frac{5}{2}) \times (-2)) \\
 & = -12
 \end{aligned}$$

A.10

$$\begin{aligned}
 & \begin{vmatrix} -1 & -2 & -1 & 1 \\ 1 & -2 & -2 & 0 \\ 1 & -2 & 1 & -1 \\ 1 & 1 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} -1 & -2 & -1 & 1 \\ 0 & -4 & -3 & 1 \\ 1 & -2 & 1 & -1 \\ 1 & 1 & 2 & 2 \end{vmatrix} \quad \text{左} \\
 & = 1 \begin{vmatrix} -1 & -2 & -1 & 1 \\ 0 & -4 & -3 & 1 \\ 0 & -4 & 0 & 0 \\ 1 & 1 & 2 & 2 \end{vmatrix} \quad \text{左} \\
 & \text{左 R(3,1;}-\frac{1}{2}) ; 3 \text{行目に} 1 \text{行目を} -\frac{1}{2} \text{倍して} \uparrow \text{加える} \quad \text{左} \\
 & = 1 \begin{vmatrix} -1 & -2 & -1 & 1 \\ 0 & -4 & -3 & 1 \\ 0 & -4 & 0 & 0 \\ 0 & -1 & 1 & 3 \end{vmatrix} \\
 & \text{左 R(4,1;1) ; 4行目に} 1 \text{行目を} 1 \text{倍して} \uparrow \text{加える} \\
 & = -1 \begin{vmatrix} -4 & -3 & 1 \\ -4 & 0 & 0 \\ -1 & 1 & 3 \\ -4 & -3 & 1 \end{vmatrix} \quad \text{左} \\
 & = -1 \begin{vmatrix} 0 & 3 & -1 \\ -1 & 1 & 3 \end{vmatrix} \\
 & \text{左 P(4,2) ; 4行目と} 2 \text{行目を} \text{交換} \\
 & = 1 \begin{vmatrix} -4 & -3 & 1 \\ 0 & 3 & -1 \\ 0 & \frac{7}{4} & \frac{11}{4} \\ 0 & 3 & -1 \end{vmatrix} \quad \text{左} \\
 & = 4 \begin{vmatrix} 3 & -1 \\ \frac{7}{4} & \frac{11}{4} \end{vmatrix} \\
 & = 4 \times (3 \times \frac{11}{4} - (-1) \times \frac{7}{4}) \\
 & = 40
 \end{aligned}$$