

# 代幾 I 計算演習 [問題] (2006/12/14)

問. 次の行列の行列式を求めなさい

Q.1

$$\begin{vmatrix} 2 & 1 & 0 & -2 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & 0 & 2 \\ 2 & -2 & 1 & 1 \end{vmatrix}$$

Q.2

$$\begin{vmatrix} 2 & 2 & -1 & -2 & -2 & 2 \\ 1 & 2 & 1 & 1 & 2 & -2 \\ -2 & -2 & 2 & 2 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -2 & 1 & -2 & -1 & 1 & 0 \\ -1 & -1 & -2 & -1 & 2 & -2 \end{vmatrix}$$

Q.3

$$\begin{vmatrix} 2 & 2 & -1 & -1 & 2 & 2 \\ -1 & 2 & 2 & 1 & 0 & 0 \\ -1 & -1 & -2 & -1 & -2 & 1 \\ 2 & 1 & 2 & -1 & 0 & -2 \\ -2 & 1 & -1 & -2 & 2 & 1 \\ -2 & -2 & 2 & 0 & 2 & 0 \end{vmatrix}$$

Q.4

$$\begin{vmatrix} 2 & 1 & 1 & -1 \\ -2 & -2 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ 0 & -1 & 1 & 2 \end{vmatrix}$$

Q.5

$$\begin{vmatrix} -2 & -1 & -1 & -2 & -2 \\ 1 & 2 & 2 & 2 & 0 \\ 0 & 2 & 1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 2 & -2 \end{vmatrix}$$

Q.6

$$\begin{vmatrix} 0 & 0 & -2 & -2 & 2 \\ 2 & -1 & -2 & -2 & 2 \\ 1 & 2 & 2 & 2 & -1 \\ 2 & -2 & 2 & -1 & -2 \\ 0 & -1 & 0 & 2 & 1 \end{vmatrix}$$

Q.7

$$\begin{vmatrix} 0 & 1 & -2 & -2 & -2 & 1 \\ -2 & -1 & -2 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 & 1 \\ 1 & -2 & 0 & -2 & -1 & -2 \\ 2 & 0 & -2 & 1 & 1 & -1 \\ 1 & -1 & -2 & 1 & -2 & 0 \end{vmatrix}$$

Q.8

$$\begin{vmatrix} -2 & -1 & -2 & -2 & 1 & -2 \\ -2 & 1 & 0 & -2 & 0 & 0 \\ -2 & -2 & -2 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & -2 & -1 \\ -2 & -1 & 1 & 2 & -1 & 1 \\ -2 & 2 & 2 & 2 & 0 & 0 \end{vmatrix}$$

Q.9

$$\begin{vmatrix} 0 & -2 & 0 & 0 & 0 \\ -2 & 2 & 2 & -2 & -1 \\ 1 & -2 & 2 & -2 & 0 \\ -1 & 1 & 0 & -2 & -1 \\ 1 & -2 & 1 & -2 & -1 \end{vmatrix}$$

Q.10

$$\begin{vmatrix} 2 & -2 & 2 & 2 & -2 \\ 0 & 0 & -1 & -1 & -1 \\ 1 & 0 & 1 & -2 & -1 \\ 2 & -1 & 2 & -1 & 1 \\ 2 & 2 & 0 & 1 & 2 \end{vmatrix}$$

# 代幾 I 計算演習 [解答] (2006/12/14)

A.1

$$\begin{vmatrix} 2 & 1 & 0 & -2 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & 0 & 2 \\ 2 & -2 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} 2 & 1 & 0 & -2 \\ 0 & -\frac{1}{2} & 1 & 1 \\ -1 & -1 & 0 & 2 \\ 2 & -2 & 1 & 1 \end{vmatrix}$$

左  $R(2,1;\frac{1}{2})$  ; 2 行目に  
1 行目を  $\frac{1}{2}$  倍して、加  
える

$$= 1 \begin{vmatrix} 2 & 1 & 0 & -2 \\ 0 & -\frac{1}{2} & 1 & 1 \\ 0 & -\frac{1}{2} & 0 & 1 \\ 2 & -2 & 1 & 1 \end{vmatrix}$$

左  $R(3,1;\frac{1}{2})$  ; 3 行目に  
1 行目を  $\frac{1}{2}$  倍して、加  
える

$$= 1 \begin{vmatrix} 2 & 1 & 0 & -2 \\ 0 & -\frac{1}{2} & 1 & 1 \\ 0 & -\frac{1}{2} & 0 & 1 \\ 0 & -3 & 1 & 3 \end{vmatrix}$$

左  $R(4,1;-1)$  ; 4 行目  
に 1 行目を  $-1$  倍して、  
加える

$$= 2 \begin{vmatrix} -\frac{1}{2} & 1 & 1 \\ -\frac{1}{2} & 0 & 1 \\ -3 & 1 & 3 \end{vmatrix}$$

$$= 2 \begin{vmatrix} -\frac{1}{2} & 1 & 1 \\ 0 & -1 & 0 \\ -3 & 1 & 3 \end{vmatrix}$$

左  $R(3,2;-1)$  ; 3 行目  
に 2 行目を  $-1$  倍して、  
加える

$$= 2 \begin{vmatrix} -\frac{1}{2} & 1 & 1 \\ 0 & -1 & 0 \\ 0 & -5 & -3 \end{vmatrix}$$

左  $R(4,2;-6)$  ; 4 行目  
に 2 行目を  $-6$  倍して、  
加える

$$= -1 \begin{vmatrix} -1 & 0 \\ -5 & -3 \end{vmatrix}$$

$$= -1 \times ((-1) \times (-3) - 0 \times (-5))$$

$$= -3$$

A.2

$$\begin{array}{l}
 \left| \begin{array}{cccccc} 2 & 2 & -1 & -2 & -2 & 2 \\ 1 & 2 & 1 & 1 & 2 & -2 \\ -2 & -2 & 2 & 2 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -2 & 1 & -2 & -1 & 1 & 0 \\ -1 & -1 & -2 & -1 & 2 & -2 \end{array} \right| = 1 \left| \begin{array}{cccccc} 2 & 2 & -1 & -2 & -2 & 2 \\ 0 & 1 & \frac{3}{2} & 2 & 3 & -3 \\ -2 & -2 & 2 & 2 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -2 & 1 & -2 & -1 & 1 & 0 \\ -1 & -1 & -2 & -1 & 2 & -2 \end{array} \right| \begin{array}{l} \text{左 } R(2,1;-\frac{1}{2}) ; 2 \text{ 行目} \\ \text{に 1 行目を } -\frac{1}{2} \text{ 倍して、} \\ \text{加える} \end{array} \\
 \\
 = 1 \left| \begin{array}{cccccc} 2 & 2 & -1 & -2 & -2 & 2 \\ 0 & 1 & \frac{3}{2} & 2 & 3 & -3 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -2 & 1 & -2 & -1 & 1 & 0 \\ -1 & -1 & -2 & -1 & 2 & -2 \end{array} \right| \begin{array}{l} \text{左 } R(3,1;1) ; 3 \text{ 行目に} \\ \text{1 行目を 1 倍して、加} \\ \text{える} \end{array} \\
 \\
 = 1 \left| \begin{array}{cccccc} 2 & 2 & -1 & -2 & -2 & 2 \\ 0 & 1 & \frac{3}{2} & 2 & 3 & -3 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & \frac{3}{2} & -1 & -1 & 1 \\ -2 & 1 & -2 & -1 & 1 & 0 \\ -1 & -1 & -2 & -1 & 2 & -2 \end{array} \right| \begin{array}{l} \text{左 } R(4,1;\frac{1}{2}) ; 4 \text{ 行目に} \\ \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\ \text{える} \end{array} \\
 \\
 = 1 \left| \begin{array}{cccccc} 2 & 2 & -1 & -2 & -2 & 2 \\ 0 & 1 & \frac{3}{2} & 2 & 3 & -3 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & \frac{3}{2} & -1 & -1 & 1 \\ 0 & 3 & -3 & -3 & -1 & 2 \\ -1 & -1 & -2 & -1 & 2 & -2 \end{array} \right| \begin{array}{l} \text{左 } R(5,1;1) ; 5 \text{ 行目に} \\ \text{1 行目を 1 倍して、加} \\ \text{える} \end{array} \\
 \\
 = 1 \left| \begin{array}{cccccc} 2 & 2 & -1 & -2 & -2 & 2 \\ 0 & 1 & \frac{3}{2} & 2 & 3 & -3 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & \frac{3}{2} & -1 & -1 & 1 \\ 0 & 3 & -3 & -3 & -1 & 2 \\ 0 & 0 & -\frac{5}{2} & -2 & 1 & -1 \end{array} \right| \begin{array}{l} \text{左 } R(6,1;\frac{1}{2}) ; 6 \text{ 行目に} \\ \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\ \text{える} \end{array} \\
 \\
 = 2 \left| \begin{array}{ccccc} 1 & \frac{3}{2} & 2 & 3 & -3 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & \frac{3}{2} & -1 & -1 & 1 \\ 3 & -3 & -3 & -1 & 2 \\ 0 & -\frac{5}{2} & -2 & 1 & -1 \end{array} \right| \\
 \\
 = 2 \left| \begin{array}{ccccc} 1 & \frac{3}{2} & 2 & 3 & -3 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & \frac{3}{2} & -1 & -1 & 1 \\ 0 & -\frac{15}{2} & -9 & -10 & 11 \\ 0 & -\frac{5}{2} & -2 & 1 & -1 \end{array} \right| \begin{array}{l} \text{左 } R(5,2;-3) ; 5 \text{ 行目} \\ \text{に 2 行目を } -3 \text{ 倍して、} \\ \text{加える} \end{array} \\
 \\
 = 2 \left| \begin{array}{cccc} 1 & 0 & -2 & 2 \\ \frac{3}{2} & -1 & -1 & 1 \end{array} \right|
 \end{array}$$



A.4

$$\begin{vmatrix} 2 & 1 & 1 & -1 \\ -2 & -2 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ 0 & -1 & 1 & 2 \end{vmatrix} = 1 \begin{vmatrix} 2 & 1 & 1 & -1 \\ 0 & -1 & 3 & -2 \\ 2 & 1 & -2 & -2 \\ 0 & -1 & 1 & 2 \end{vmatrix}$$

左 R(2,1;1) ; 2行目に  
1行目を1倍して、加  
える

$$= 1 \begin{vmatrix} 2 & 1 & 1 & -1 \\ 0 & -1 & 3 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & -1 & 1 & 2 \end{vmatrix}$$

左 R(3,1;-1) ; 3行目  
に1行目を-1倍して、  
加える

$$= 2 \begin{vmatrix} -1 & 3 & -2 \\ 0 & -3 & -1 \\ -1 & 1 & 2 \\ -1 & 3 & -2 \\ 0 & -3 & -1 \\ 0 & -2 & 4 \end{vmatrix}$$

左 R(4,2;-1) ; 4行目  
に2行目を-1倍して、  
加える

$$= -2 \begin{vmatrix} -3 & -1 \\ -2 & 4 \end{vmatrix}$$

$$= -2 \times ((-3) \times 4 - (-1) \times (-2))$$

$$= 28$$

$$\begin{aligned}
& \left| \begin{array}{ccccc} -2 & -1 & -1 & -2 & -2 \\ 1 & 2 & 2 & 2 & 0 \\ 0 & 2 & 1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 2 & -2 \end{array} \right| = 1 \left| \begin{array}{ccccc} -2 & -1 & -1 & -2 & -2 \\ 0 & \frac{3}{2} & \frac{3}{2} & 1 & -1 \\ 0 & 2 & 1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 2 & -2 \end{array} \right| & \text{左 } R(2,1;\frac{1}{2}) ; 2 \text{ 行目に} \\
& & & & \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\
& & & & \text{える} \\
& = 1 \left| \begin{array}{ccccc} -2 & -1 & -1 & -2 & -2 \\ 0 & \frac{3}{2} & \frac{3}{2} & 1 & -1 \\ 0 & 2 & 1 & -1 & 1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 2 \\ -1 & -1 & -1 & 2 & -2 \end{array} \right| & \text{左 } R(4,1;-\frac{1}{2}) ; 4 \text{ 行目} \\
& & & & \text{に 1 行目を } -\frac{1}{2} \text{ 倍して、} \\
& & & & \text{加える} \\
& = 1 \left| \begin{array}{ccccc} -2 & -1 & -1 & -2 & -2 \\ 0 & \frac{3}{2} & \frac{3}{2} & 1 & -1 \\ 0 & 2 & 1 & -1 & 1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 2 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 3 & -1 \end{array} \right| & \text{左 } R(5,1;-\frac{1}{2}) ; 5 \text{ 行目} \\
& & & & \text{に 1 行目を } -\frac{1}{2} \text{ 倍して、} \\
& & & & \text{加える} \\
& = -2 \left| \begin{array}{ccccc} \frac{3}{2} & \frac{3}{2} & 1 & -1 \\ 2 & 1 & -1 & 1 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & 2 \\ -\frac{1}{2} & -\frac{1}{2} & 3 & -1 \\ \frac{3}{2} & \frac{3}{2} & 1 & -1 \end{array} \right| & \\
& = -2 \left| \begin{array}{ccccc} \frac{3}{2} & \frac{3}{2} & 1 & -1 \\ 0 & -1 & -\frac{7}{3} & \frac{7}{3} \\ -\frac{1}{2} & -\frac{1}{2} & 0 & 2 \\ -\frac{1}{2} & -\frac{1}{2} & 3 & -1 \end{array} \right| & \text{左 } R(3,2;-\frac{4}{3}) ; 3 \text{ 行目} \\
& & & & \text{に 2 行目を } -\frac{4}{3} \text{ 倍して、} \\
& & & & \text{加える} \\
& = -2 \left| \begin{array}{ccccc} \frac{3}{2} & \frac{3}{2} & 1 & -1 \\ 0 & -1 & -\frac{7}{3} & \frac{7}{3} \\ 0 & 0 & \frac{1}{3} & \frac{5}{3} \\ -\frac{1}{2} & -\frac{1}{2} & 3 & -1 \end{array} \right| & \text{左 } R(4,2;\frac{1}{3}) ; 4 \text{ 行目に} \\
& & & & \text{2 行目を } \frac{1}{3} \text{ 倍して、加} \\
& & & & \text{える} \\
& = -2 \left| \begin{array}{ccccc} \frac{3}{2} & \frac{3}{2} & 1 & -1 \\ 0 & -1 & -\frac{7}{3} & \frac{7}{3} \\ 0 & 0 & \frac{1}{3} & \frac{5}{3} \\ 0 & 0 & \frac{10}{3} & -\frac{4}{3} \end{array} \right| & \text{左 } R(5,2;\frac{1}{3}) ; 5 \text{ 行目に} \\
& & & & \text{2 行目を } \frac{1}{3} \text{ 倍して、加} \\
& & & & \text{える} \\
& = -3 \left| \begin{array}{ccc} -1 & -\frac{7}{3} & \frac{7}{3} \\ 0 & \frac{1}{3} & \frac{5}{3} \\ 0 & \frac{10}{3} & -\frac{4}{3} \end{array} \right| \\
& = 3 \left| \begin{array}{cc} \frac{1}{3} & \frac{5}{3} \\ \frac{10}{3} & -\frac{4}{3} \end{array} \right| \\
& = 3 \times (\frac{1}{3} \times (-\frac{4}{3}) - \frac{5}{3} \times \frac{10}{3}) \\
& = -18
\end{aligned}$$

$$\begin{aligned}
& \left| \begin{array}{ccccc} 0 & 0 & -2 & -2 & 2 \\ 2 & -1 & -2 & -2 & 2 \\ 1 & 2 & 2 & 2 & -1 \\ 2 & -2 & 2 & -1 & -2 \\ 0 & -1 & 0 & 2 & 1 \end{array} \right| = -1 \left| \begin{array}{ccccc} 2 & -1 & -2 & -2 & 2 \\ 0 & 0 & -2 & -2 & 2 \\ 1 & 2 & 2 & 2 & -1 \\ 2 & -2 & 2 & -1 & -2 \\ 0 & -1 & 0 & 2 & 1 \end{array} \right| & \text{左 } P(2,1) ; 2 \text{ 行目と } 1 \text{ 行目を交換} \\
& = -1 \left| \begin{array}{ccccc} 2 & -1 & -2 & -2 & 2 \\ 0 & 0 & -2 & -2 & 2 \\ 0 & \frac{5}{2} & 3 & 3 & -2 \\ 2 & -2 & 2 & -1 & -2 \\ 0 & -1 & 0 & 2 & 1 \end{array} \right| & \text{左 } R(3,1;-\frac{1}{2}) ; 3 \text{ 行目} \\
& & & & \text{に } 1 \text{ 行目を } -\frac{1}{2} \text{ 倍して、} \\
& & & & \text{加える} \\
& = -1 \left| \begin{array}{ccccc} 2 & -1 & -2 & -2 & 2 \\ 0 & 0 & -2 & -2 & 2 \\ 0 & \frac{5}{2} & 3 & 3 & -2 \\ 0 & -1 & 4 & 1 & -4 \\ 0 & -1 & 0 & 2 & 1 \end{array} \right| & \text{左 } R(4,1;-1) ; 4 \text{ 行目} \\
& & & & \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\
& & & & \text{加える} \\
& = -2 \left| \begin{array}{ccccc} 0 & -2 & -2 & 2 \\ \frac{5}{2} & 3 & 3 & -2 \\ -1 & 4 & 1 & -4 \\ -1 & 0 & 2 & 1 \end{array} \right| \\
& = 2 \left| \begin{array}{ccccc} \frac{5}{2} & 3 & 3 & -2 \\ 0 & -2 & -2 & 2 \\ -1 & 4 & 1 & -4 \\ -1 & 0 & 2 & 1 \end{array} \right| & \text{左 } P(3,2) ; 3 \text{ 行目と } 2 \text{ 行目を交換} \\
& = 2 \left| \begin{array}{ccccc} \frac{5}{2} & 3 & 3 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & \frac{26}{5} & \frac{11}{5} & -\frac{24}{5} \\ -1 & 0 & 2 & 1 \end{array} \right| & \text{左 } R(4,2;\frac{2}{5}) ; 4 \text{ 行目に} \\
& & & & \text{ } 2 \text{ 行目を } \frac{2}{5} \text{ 倍して、加} \\
& & & & \text{える} \\
& = 2 \left| \begin{array}{ccccc} \frac{5}{2} & 3 & 3 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & \frac{26}{5} & \frac{11}{5} & -\frac{24}{5} \\ 0 & \frac{6}{5} & \frac{16}{5} & \frac{1}{5} \end{array} \right| & \text{左 } R(5,2;\frac{2}{5}) ; 5 \text{ 行目に} \\
& & & & \text{ } 2 \text{ 行目を } \frac{2}{5} \text{ 倍して、加} \\
& & & & \text{える} \\
& = 5 \left| \begin{array}{ccc} -2 & -2 & 2 \\ \frac{26}{5} & \frac{11}{5} & -\frac{24}{5} \\ \frac{6}{5} & \frac{16}{5} & \frac{1}{5} \end{array} \right| \\
& = 5 \left| \begin{array}{ccc} -2 & -2 & 2 \\ 0 & -3 & \frac{2}{5} \\ \frac{6}{5} & \frac{16}{5} & \frac{1}{5} \end{array} \right| & \text{左 } R(4,3;\frac{13}{5}) ; 4 \text{ 行目に} \\
& & & & \text{ } 3 \text{ 行目を } \frac{13}{5} \text{ 倍して、加} \\
& & & & \text{える} \\
& = 5 \left| \begin{array}{ccc} -2 & -2 & 2 \\ 0 & -3 & \frac{2}{5} \\ 0 & 2 & \frac{7}{5} \end{array} \right| & \text{左 } R(5,3;\frac{3}{5}) ; 5 \text{ 行目に} \\
& & & & \text{ } 3 \text{ 行目を } \frac{3}{5} \text{ 倍して、加} \\
& & & & \text{える} \\
& = -10 \left| \begin{array}{c} -3 \\ \frac{2}{5} \end{array} \right|
\end{aligned}$$

A.7

$$\begin{vmatrix} 0 & 1 & -2 & -2 & -2 & 1 \\ -2 & -1 & -2 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 & 1 \\ 1 & -2 & 0 & -2 & -1 & -2 \\ 2 & 0 & -2 & 1 & 1 & -1 \\ 1 & -1 & -2 & 1 & -2 & 0 \end{vmatrix} = -1 \begin{vmatrix} -2 & -1 & -2 & -2 & 1 & 0 \\ 0 & 1 & -2 & -2 & -2 & 1 \\ 0 & -2 & 1 & 0 & 1 & 1 \\ 1 & -2 & 0 & -2 & -1 & -2 \\ 2 & 0 & -2 & 1 & 1 & -1 \\ 1 & -1 & -2 & 1 & -2 & 0 \end{vmatrix}$$

左 P(2,1) ; 2行目と1行目を交換

$$= -1 \begin{vmatrix} -2 & -1 & -2 & -2 & 1 & 0 \\ 0 & 1 & -2 & -2 & -2 & 1 \\ 0 & -2 & 1 & 0 & 1 & 1 \\ 0 & -\frac{5}{2} & -1 & -3 & -\frac{1}{2} & -2 \\ 2 & 0 & -2 & 1 & 1 & -1 \\ 1 & -1 & -2 & 1 & -2 & 0 \end{vmatrix}$$

左 R(4,1; $\frac{1}{2}$ ) ; 4行目に1行目を $\frac{1}{2}$ 倍して、加える

$$= -1 \begin{vmatrix} -2 & -1 & -2 & -2 & 1 & 0 \\ 0 & 1 & -2 & -2 & -2 & 1 \\ 0 & -2 & 1 & 0 & 1 & 1 \\ 0 & -\frac{5}{2} & -1 & -3 & -\frac{1}{2} & -2 \\ 0 & -1 & -4 & -1 & 2 & -1 \\ 1 & -1 & -2 & 1 & -2 & 0 \end{vmatrix}$$

左 R(5,1;1) ; 5行目に1行目を1倍して、加える

$$= -1 \begin{vmatrix} -2 & -1 & -2 & -2 & 1 & 0 \\ 0 & 1 & -2 & -2 & -2 & 1 \\ 0 & -2 & 1 & 0 & 1 & 1 \\ 0 & -\frac{5}{2} & -1 & -3 & -\frac{1}{2} & -2 \\ 0 & -1 & -4 & -1 & 2 & -1 \\ 0 & -\frac{3}{2} & -3 & 0 & -\frac{3}{2} & 0 \end{vmatrix}$$

左 R(6,1; $\frac{1}{2}$ ) ; 6行目に1行目を $\frac{1}{2}$ 倍して、加える

$$= 2 \begin{vmatrix} 1 & -2 & -2 & -2 & 1 \\ -2 & 1 & 0 & 1 & 1 \\ -\frac{5}{2} & -1 & -3 & -\frac{1}{2} & -2 \\ -1 & -4 & -1 & 2 & -1 \\ -\frac{3}{2} & -3 & 0 & -\frac{3}{2} & 0 \\ 1 & -2 & -2 & -2 & 1 \\ 0 & -3 & -4 & -3 & 3 \\ -\frac{5}{2} & -1 & -3 & -\frac{1}{2} & -2 \\ -1 & -4 & -1 & 2 & -1 \\ -\frac{3}{2} & -3 & 0 & -\frac{3}{2} & 0 \end{vmatrix}$$

左 R(3,2;2) ; 3行目に2行目を2倍して、加える

$$= 2 \begin{vmatrix} 1 & -2 & -2 & -2 & 1 \\ 0 & -3 & -4 & -3 & 3 \\ 0 & -6 & -8 & -\frac{11}{2} & \frac{1}{2} \\ -1 & -4 & -1 & 2 & -1 \\ -\frac{3}{2} & -3 & 0 & -\frac{3}{2} & 0 \end{vmatrix}$$

左 R(4,2; $\frac{5}{2}$ ) ; 4行目に2行目を $\frac{5}{2}$ 倍して、加える

$$= 2 \begin{vmatrix} 1 & -2 & -2 & -2 & 1 \\ 0 & -3 & -4 & -3 & 3 \\ 0 & -6 & -8 & -\frac{11}{2} & \frac{1}{2} \\ 0 & -6 & -8 & -\frac{11}{2} & \frac{1}{2} \\ 0 & -6 & -8 & -\frac{11}{2} & \frac{1}{2} \end{vmatrix}$$

左 R(5,2;1) ; 5行目に2行目を1倍して、加える



$$\begin{array}{l}
\left| \begin{array}{cccccc} -2 & -1 & -2 & -2 & 1 & -2 \\ -2 & 1 & 0 & -2 & 0 & 0 \\ -2 & -2 & -2 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & -2 & -1 \\ -2 & -1 & 1 & 2 & -1 & 1 \\ -2 & 2 & 2 & 2 & 0 & 0 \end{array} \right| \\
= 1 \left| \begin{array}{cccccc} -2 & -1 & -2 & -2 & 1 & -2 \\ 0 & 2 & 2 & 0 & -1 & 2 \\ -2 & -2 & -2 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & -2 & -1 \\ -2 & -1 & 1 & 2 & -1 & 1 \\ -2 & 2 & 2 & 2 & 0 & 0 \end{array} \right| \quad \begin{array}{l} \text{左 } R(2,1;-1) ; 2 \text{ 行目} \\ \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\ \text{加える} \end{array} \\
= 1 \left| \begin{array}{cccccc} -2 & -1 & -2 & -2 & 1 & -2 \\ 0 & 2 & 2 & 0 & -1 & 2 \\ 0 & -1 & 0 & 2 & -1 & 1 \\ 1 & -1 & 0 & 1 & -2 & -1 \\ -2 & -1 & 1 & 2 & -1 & 1 \\ -2 & 2 & 2 & 2 & 0 & 0 \end{array} \right| \quad \begin{array}{l} \text{左 } R(3,1;-1) ; 3 \text{ 行目} \\ \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\ \text{加える} \end{array} \\
= 1 \left| \begin{array}{cccccc} -2 & -1 & -2 & -2 & 1 & -2 \\ 0 & 2 & 2 & 0 & -1 & 2 \\ 0 & -1 & 0 & 2 & -1 & 1 \\ 0 & -\frac{3}{2} & -1 & 0 & -\frac{3}{2} & -2 \\ -2 & -1 & 1 & 2 & -1 & 1 \\ -2 & 2 & 2 & 2 & 0 & 0 \end{array} \right| \quad \begin{array}{l} \text{左 } R(4,1;\frac{1}{2}) ; 4 \text{ 行目に} \\ 1 \text{ 行目を } \frac{1}{2} \text{ 倍して、加} \\ \text{える} \end{array} \\
= 1 \left| \begin{array}{cccccc} -2 & -1 & -2 & -2 & 1 & -2 \\ 0 & 2 & 2 & 0 & -1 & 2 \\ 0 & -1 & 0 & 2 & -1 & 1 \\ 0 & -\frac{3}{2} & -1 & 0 & -\frac{3}{2} & -2 \\ 0 & 0 & 3 & 4 & -2 & 3 \\ -2 & 2 & 2 & 2 & 0 & 0 \end{array} \right| \quad \begin{array}{l} \text{左 } R(5,1;-1) ; 5 \text{ 行目} \\ \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\ \text{加える} \end{array} \\
= 1 \left| \begin{array}{cccccc} -2 & -1 & -2 & -2 & 1 & -2 \\ 0 & 2 & 2 & 0 & -1 & 2 \\ 0 & -1 & 0 & 2 & -1 & 1 \\ 0 & -\frac{3}{2} & -1 & 0 & -\frac{3}{2} & -2 \\ 0 & 0 & 3 & 4 & -2 & 3 \\ 0 & 3 & 4 & 4 & -1 & 2 \end{array} \right| \quad \begin{array}{l} \text{左 } R(6,1;-1) ; 6 \text{ 行目} \\ \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\ \text{加える} \end{array} \\
= -2 \left| \begin{array}{cccccc} 2 & 2 & 0 & -1 & 2 \\ -1 & 0 & 2 & -1 & 1 \\ -\frac{3}{2} & -1 & 0 & -\frac{3}{2} & -2 \\ 0 & 3 & 4 & -2 & 3 \\ 3 & 4 & 4 & -1 & 2 \end{array} \right| \\
= -2 \left| \begin{array}{cccccc} 2 & 2 & 0 & -1 & 2 \\ 0 & 1 & 2 & -\frac{3}{2} & 2 \\ -\frac{3}{2} & -1 & 0 & -\frac{3}{2} & -2 \\ 0 & 3 & 4 & -2 & 3 \\ 3 & 4 & 4 & -1 & 2 \end{array} \right| \quad \begin{array}{l} \text{左 } R(3,2;\frac{1}{2}) ; 3 \text{ 行目に} \\ 2 \text{ 行目を } \frac{1}{2} \text{ 倍して、加} \\ \text{える} \end{array} \\
\left| \begin{array}{cccccc} 2 & 2 & 0 & -1 & 2 \\ 0 & 1 & 2 & -\frac{3}{2} & 2 \\ 1 & 1 & 2 & -\frac{3}{2} & 1 \end{array} \right| \quad \begin{array}{l} \text{左 } R(4,2;\frac{3}{4}) ; 4 \text{ 行目に} \\ \text{2 行目を } \frac{3}{4} \text{ 倍して、加} \\ \text{える} \end{array}
\end{array}$$

A.9

$$\begin{vmatrix} 0 & -2 & 0 & 0 & 0 \\ -2 & 2 & 2 & -2 & -1 \\ 1 & -2 & 2 & -2 & 0 \\ -1 & 1 & 0 & -2 & -1 \\ 1 & -2 & 1 & -2 & -1 \end{vmatrix} = -1 \begin{vmatrix} -2 & 2 & 2 & -2 & -1 \\ 0 & -2 & 0 & 0 & 0 \\ 1 & -2 & 2 & -2 & 0 \\ -1 & 1 & 0 & -2 & -1 \\ 1 & -2 & 1 & -2 & -1 \end{vmatrix}$$

左 P(2,1) ; 2行目と1行目を交換

$$= -1 \begin{vmatrix} -2 & 2 & 2 & -2 & -1 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & 3 & -3 & -\frac{1}{2} \\ -1 & 1 & 0 & -2 & -1 \\ 1 & -2 & 1 & -2 & -1 \end{vmatrix}$$

左 R(3,1; $\frac{1}{2}$ ) ; 3行目に1行目を $\frac{1}{2}$ 倍して、加える

$$= -1 \begin{vmatrix} -2 & 2 & 2 & -2 & -1 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & 3 & -3 & -\frac{1}{2} \\ 0 & 0 & -1 & -1 & -\frac{1}{2} \\ 1 & -2 & 1 & -2 & -1 \end{vmatrix}$$

左 R(4,1; $-\frac{1}{2}$ ) ; 4行目に1行目を $-\frac{1}{2}$ 倍して、加える

$$= -1 \begin{vmatrix} -2 & 2 & 2 & -2 & -1 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & 3 & -3 & -\frac{1}{2} \\ 0 & 0 & -1 & -1 & -\frac{1}{2} \\ 0 & -1 & 2 & -3 & -\frac{3}{2} \end{vmatrix}$$

左 R(5,1; $\frac{1}{2}$ ) ; 5行目に1行目を $\frac{1}{2}$ 倍して、加える

$$= 2 \begin{vmatrix} -2 & 0 & 0 & 0 \\ -1 & 3 & -3 & -\frac{1}{2} \\ 0 & -1 & -1 & -\frac{1}{2} \\ -1 & 2 & -3 & -\frac{3}{2} \\ -2 & 0 & 0 & 0 \\ 0 & 3 & -3 & -\frac{1}{2} \\ 0 & -1 & -1 & -\frac{1}{2} \\ -1 & 2 & -3 & -\frac{3}{2} \end{vmatrix}$$

左 R(3,2; $-\frac{1}{2}$ ) ; 3行目に2行目を $-\frac{1}{2}$ 倍して、加える

$$= 2 \begin{vmatrix} -2 & 0 & 0 & 0 \\ 0 & 3 & -3 & -\frac{1}{2} \\ 0 & -1 & -1 & -\frac{1}{2} \\ 0 & 2 & -3 & -\frac{3}{2} \end{vmatrix}$$

左 R(5,2; $-\frac{1}{2}$ ) ; 5行目に2行目を $-\frac{1}{2}$ 倍して、加える

$$= -4 \begin{vmatrix} 3 & -3 & -\frac{1}{2} \\ -1 & -1 & -\frac{1}{2} \\ 2 & -3 & -\frac{3}{2} \\ 3 & -3 & -\frac{1}{2} \\ 0 & -2 & -\frac{2}{3} \\ 2 & -3 & -\frac{3}{2} \end{vmatrix}$$

左 R(4,3; $\frac{1}{3}$ ) ; 4行目に3行目を $\frac{1}{3}$ 倍して、加える

$$= -4 \begin{vmatrix} 3 & -3 & -\frac{1}{2} \\ 0 & -2 & -\frac{2}{3} \\ 0 & -1 & -\frac{7}{6} \end{vmatrix}$$

左 R(5,3; $-\frac{2}{3}$ ) ; 5行目に3行目を $-\frac{2}{3}$ 倍して、加える

A.10

$$\begin{aligned}
 & \begin{vmatrix} 2 & -2 & 2 & 2 & -2 \\ 0 & 0 & -1 & -1 & -1 \\ 1 & 0 & 1 & -2 & -1 \\ 2 & -1 & 2 & -1 & 1 \\ 2 & 2 & 0 & 1 & 2 \end{vmatrix} = 1 \begin{vmatrix} 2 & -2 & 2 & 2 & -2 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & -3 & 0 \\ 2 & -1 & 2 & -1 & 1 \\ 2 & 2 & 0 & 1 & 2 \end{vmatrix} & \text{左 } R(3,1;-\frac{1}{2}) ; 3 \text{ 行目} \\
 & & \text{に } 1 \text{ 行目を } -\frac{1}{2} \text{ 倍して、} \\
 & & \text{加える} \\
 & = 1 \begin{vmatrix} 2 & -2 & 2 & 2 & -2 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -3 & 3 \\ 2 & 2 & 0 & 1 & 2 \end{vmatrix} & \text{左 } R(4,1;-1) ; 4 \text{ 行目} \\
 & & \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\
 & & \text{加える} \\
 & = 1 \begin{vmatrix} 2 & -2 & 2 & 2 & -2 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 4 & -2 & -1 & 4 \end{vmatrix} & \text{左 } R(5,1;-1) ; 5 \text{ 行目} \\
 & & \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\
 & & \text{加える} \\
 & = 2 \begin{vmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & -3 & 0 \\ 1 & 0 & -3 & 3 \\ 4 & -2 & -1 & 4 \end{vmatrix} \\
 & = -2 \begin{vmatrix} 1 & 0 & -3 & 0 \\ 0 & -1 & -1 & -1 \\ 1 & 0 & -3 & 3 \\ 4 & -2 & -1 & 4 \end{vmatrix} & \text{左 } P(3,2) ; 3 \text{ 行目と } 2 \text{ 行目を交換} \\
 & = -2 \begin{vmatrix} 1 & 0 & -3 & 0 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 3 \\ 4 & -2 & -1 & 4 \end{vmatrix} & \text{左 } R(4,2;-1) ; 4 \text{ 行目} \\
 & & \text{に } 2 \text{ 行目を } -1 \text{ 倍して、} \\
 & & \text{加える} \\
 & = -2 \begin{vmatrix} 1 & 0 & -3 & 0 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & -2 & 11 & 4 \end{vmatrix} & \text{左 } R(5,2;-4) ; 5 \text{ 行目} \\
 & & \text{に } 2 \text{ 行目を } -4 \text{ 倍して、} \\
 & & \text{加える} \\
 & = -2 \begin{vmatrix} -1 & -1 & -1 \\ 0 & 0 & 3 \\ -2 & 11 & 4 \\ -1 & -1 & -1 \\ 0 & 0 & 3 \\ 0 & 13 & 6 \end{vmatrix} & \text{左 } R(5,3;-2) ; 5 \text{ 行目} \\
 & & \text{に } 3 \text{ 行目を } -2 \text{ 倍して、} \\
 & & \text{加える} \\
 & = 2 \begin{vmatrix} 0 & 3 \\ 13 & 6 \end{vmatrix} \\
 & = 2 \times (0 \times 6 - 3 \times 13) \\
 & = -78
 \end{aligned}$$