

代幾 I 計算演習 (2007/04/19)

問 1. $\alpha = 1 + i, \beta = 2 - 3i$ として、次の計算をなさい。

1. Q. $\alpha + \beta$

A.

$$\begin{aligned}\alpha + \beta &= (1 + i) + (2 - 3i) \\ &= (1 + 1) + (2 + (-3))i \\ &= (1 + 2) + (1 + (-3))i \\ &= 3 + (-2)i \\ &= 3 - 2i\end{aligned}$$

2. Q. $\alpha - \beta$

A.

$$\begin{aligned}\alpha - \beta &= (1 + i) - (2 - 3i) \\ &= (1 + 1) - (2 + (-3))i \\ &= (1 - 2) + (1 - (-3))i \\ &= (-1) + (4)i \\ &= -1 + 4i\end{aligned}$$

3. Q. $\alpha \times \beta$

A.

$$\begin{aligned}\alpha \times \beta &= (1 + i) \times (2 - 3i) \\ &= 1 \times (2 - 3i) + i \times (2 - 3i) \\ &= (1 \times 2) - (1 \times 3i) + (i \times 2) - (i \times 3i) \\ &= 2 - 3i + 2i - 3i^2 \\ &= 2 - 3i + 2i - 3(-1) \\ &= 2 - 3i + 2i + 3 \\ &= (2 + 3) + (-3i + 2i) \\ &= (2 + 3) + (-3 + 2)i \\ &= 5 + (-1)i \\ &= 5 - i\end{aligned}$$

4. Q. $\frac{\alpha}{\beta}$

A.

$$\begin{aligned}\frac{\alpha}{\beta} &= \frac{(1+i)}{(2-3i)} \\ &= \frac{(1+i)(2-3i)}{(2-3i)(2-3i)} \\ &= \frac{(1+i)(2+3i)}{(2-3i)(2+3i)} \\ &= \frac{(2+3i+2i-3)}{2^2+(-3)^2} \\ &= \frac{-1+5i}{13} \\ &= -\frac{1}{13} + \frac{5}{13}i\end{aligned}$$

5. Q. $\operatorname{Re}(\beta)$

A.

$$\begin{aligned}\operatorname{Re}(\beta) &= \operatorname{Re}(2-3i) \\ &= 2\end{aligned}$$

6. Q. $\operatorname{Im}(\beta)$

A.

$$\begin{aligned}\operatorname{Im}(\beta) &= \operatorname{Im}(2-3i) \\ &= -3\end{aligned}$$

7. Q. $\bar{\beta}$

A.

$$\begin{aligned}\bar{\beta} &= \overline{2-3i} \\ &= 2+3i\end{aligned}$$

8. Q. $|\beta|$

A.

$$\begin{aligned}|\beta| &= \sqrt{2^2+3^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13}\end{aligned}$$

問 2. 次の α, β に対して、問 1 と同様の計算を行いなさい。

1. **Q.** $\alpha = 1 + 5i$, $\beta = 1 - 3i$

A. 1-1. $2 + 2i$, 1-2. $8i$, 1-3. $16 + 2i$, 1-4. $\frac{-14+8i}{10}$, 1-5. 1 , 1-6. -3 , 1-7. $1 + 3i$, 1-8. $\sqrt{10}$

2. **Q.** $\alpha = -4 - i$, $\beta = -5 + i$

A. 2-1. -9 , 2-2. $1 - 2i$, 2-3. $21 + i$, 2-4. $\frac{19+9i}{26}$, 2-5. -5 , 2-6. 1 , 2-7. $-5 - i$, 2-8. $\sqrt{26}$

3. **Q.** $\alpha = -2 - 4i$, $\beta = 3 + 2i$

A. 3-1. $1 - 2i$, 3-2. $-5 - 6i$, 3-3. $2 - 16i$, 3-4. $\frac{-14-8i}{13}$, 3-5. 3 , 3-6. 2 , 3-7. $3 - 2i$, 3-8. $\sqrt{13}$

4. **Q.** $\alpha = -2i$, $\beta = 2 - i$

A. 4-1. $2 - 3i$, 4-2. $-2 - i$, 4-3. $-2 - 4i$, 4-4. $\frac{2-4i}{5}$, 4-5. 2 , 4-6. -1 , 4-7. $2 + i$, 4-8. $\sqrt{5}$

5. **Q.** $\alpha = 4 + 5i$, $\beta = -3 - 5i$

A. 5-1. 1 , 5-2. $7 + 10i$, 5-3. $13 - 35i$, 5-4. $\frac{-37+5i}{34}$, 5-5. -3 , 5-6. -5 , 5-7. $-3 + 5i$, 5-8. $\sqrt{34}$

6. **Q.** $\alpha = 5 + 3i$, $\beta = -5i$

A. 6-1. $5 - 2i$, 6-2. $5 + 8i$, 6-3. $15 - 25i$, 6-4. $\frac{-15+25i}{25}$, 6-5. 0 , 6-6. -5 , 6-7. $5i$, 6-8. $\sqrt{25}$

7. **Q.** $\alpha = -1 + i$, $\beta = -5 + 5i$

A. 7-1. $-6 + 6i$, 7-2. $4 - 4i$, 7-3. $-10i$, 7-4. $\frac{10}{50}$, 7-5. -5 , 7-6. 5 , 7-7. $-5 - 5i$, 7-8. $\sqrt{50}$