

代幾 I 計算演習 [問題] (2007/05/10)

問. 次の複素数の指定された自然数の冪乗根を求めなさい

Q.1

$-3i$ の 2 乗根

Q.9

-4 の 2 乗根

Q.2

$-\frac{16\sqrt{2}}{27}i$ の 3 乗根

Q.10

$\frac{2\sqrt{3}}{9} - \frac{2\sqrt{3}}{9}i$ の 3 乗根

Q.3

$-\frac{\sqrt{2}}{4}$ の 3 乗根

Q.11

$-2 + 2i$ の 3 乗根

Q.4

$-\frac{\sqrt{2}}{4}i$ の 3 乗根

Q.12

$-\frac{27}{2} + \frac{9\sqrt{3}}{2}i$ の 5 乗根

Q.5

$-\frac{1}{4} + \frac{i}{4}$ の 3 乗根

Q.13

$-\frac{9\sqrt{6}}{16} + \frac{27\sqrt{2}}{16}i$ の 5 乗根

Q.6

$-\frac{19683}{2} + \frac{19683\sqrt{3}}{2}i$ の 6 乗根

Q.14

-2 の 2 乗根

Q.7

$-\frac{27\sqrt{2}}{128} + \frac{27\sqrt{2}}{128}i$ の 6 乗根

Q.15

$-324i$ の 4 乗根

Q.8

$-\frac{2\sqrt{2}}{81} - \frac{2\sqrt{2}}{81}i$ の 4 乗根

Q.16

$-2 + 2i$ の 3 乗根

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A.1

$$-\frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2}i, \quad \frac{\sqrt{6}}{2} - \frac{\sqrt{6}}{2}i$$

A.2

$$\frac{2\sqrt{2}}{3}i, \quad -\frac{\sqrt{6}}{3} - \frac{\sqrt{2}}{3}i, \quad \frac{\sqrt{6}}{3} - \frac{\sqrt{2}}{3}i$$

A.3

$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}i, \quad -\frac{\sqrt{2}}{2}, \quad \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}i$$

A.4

$$\frac{\sqrt{2}}{2}i, \quad -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}i, \quad \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}i$$

A.5

$$\frac{1}{2} + \frac{i}{2}, \quad \frac{\sqrt{2}}{2}(\cos \frac{11}{12}\pi + i \sin \frac{11}{12}\pi), \quad \frac{\sqrt{2}}{2}(\cos \frac{19}{12}\pi + i \sin \frac{19}{12}\pi)$$

A.6

$$3\sqrt{3}(\cos \frac{\pi}{9} + i \sin \frac{\pi}{9}), \quad 3\sqrt{3}(\cos \frac{4}{9}\pi + i \sin \frac{4}{9}\pi), \quad 3\sqrt{3}(\cos \frac{7}{9}\pi + i \sin \frac{7}{9}\pi), \\ 3\sqrt{3}(\cos \frac{10}{9}\pi + i \sin \frac{10}{9}\pi), \quad 3\sqrt{3}(\cos \frac{13}{9}\pi + i \sin \frac{13}{9}\pi), \quad 3\sqrt{3}(\cos \frac{16}{9}\pi + i \sin \frac{16}{9}\pi)$$

A.7

$$\frac{\sqrt{3}}{2}(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}), \quad \frac{\sqrt{3}}{2}(\cos \frac{11}{24}\pi + i \sin \frac{11}{24}\pi), \quad \frac{\sqrt{3}}{2}(\cos \frac{19}{24}\pi + i \sin \frac{19}{24}\pi), \\ \frac{\sqrt{3}}{2}(\cos \frac{9}{8}\pi + i \sin \frac{9}{8}\pi), \quad \frac{\sqrt{3}}{2}(\cos \frac{35}{24}\pi + i \sin \frac{35}{24}\pi), \quad \frac{\sqrt{3}}{2}(\cos \frac{43}{24}\pi + i \sin \frac{43}{24}\pi)$$

A.8

$$\frac{\sqrt{2}}{3}(\cos \frac{5}{16}\pi + i \sin \frac{5}{16}\pi), \quad \frac{\sqrt{2}}{3}(\cos \frac{13}{16}\pi + i \sin \frac{13}{16}\pi), \quad \frac{\sqrt{2}}{3}(\cos \frac{21}{16}\pi + i \sin \frac{21}{16}\pi), \\ \frac{\sqrt{2}}{3}(\cos \frac{29}{16}\pi + i \sin \frac{29}{16}\pi)$$

A.9

$$2i, -2i$$

A.10

$$\frac{\sqrt{6}}{3}(\cos \frac{7}{12}\pi + i \sin \frac{7}{12}\pi), -\frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3}i, \frac{\sqrt{6}}{3}(\cos \frac{23}{12}\pi + i \sin \frac{23}{12}\pi)$$

A.11

$$1 + i, \sqrt{2}(\cos \frac{11}{12}\pi + i \sin \frac{11}{12}\pi), \sqrt{2}(\cos \frac{19}{12}\pi + i \sin \frac{19}{12}\pi)$$

A.12

$$\frac{3}{2} + \frac{\sqrt{3}}{2}i, \sqrt{3}(\cos \frac{17}{30}\pi + i \sin \frac{17}{30}\pi), \sqrt{3}(\cos \frac{29}{30}\pi + i \sin \frac{29}{30}\pi),$$
$$\sqrt{3}(\cos \frac{41}{30}\pi + i \sin \frac{41}{30}\pi), \sqrt{3}(\cos \frac{53}{30}\pi + i \sin \frac{53}{30}\pi)$$

A.13

$$\frac{\sqrt{6}}{2}(\cos \frac{2}{15}\pi + i \sin \frac{2}{15}\pi), \frac{\sqrt{6}}{2}(\cos \frac{8}{15}\pi + i \sin \frac{8}{15}\pi), \frac{\sqrt{6}}{2}(\cos \frac{14}{15}\pi + i \sin \frac{14}{15}\pi),$$
$$-\frac{\sqrt{6}}{4} - \frac{3\sqrt{2}}{4}i, \frac{\sqrt{6}}{2}(\cos \frac{26}{15}\pi + i \sin \frac{26}{15}\pi)$$

A.14

$$\sqrt{2}i, -\sqrt{2}i$$

A.15

$$3\sqrt{2}(\cos \frac{3}{8}\pi + i \sin \frac{3}{8}\pi), 3\sqrt{2}(\cos \frac{7}{8}\pi + i \sin \frac{7}{8}\pi), 3\sqrt{2}(\cos \frac{11}{8}\pi + i \sin \frac{11}{8}\pi),$$
$$3\sqrt{2}(\cos \frac{15}{8}\pi + i \sin \frac{15}{8}\pi)$$

A.16

$$1 + i, \sqrt{2}(\cos \frac{11}{12}\pi + i \sin \frac{11}{12}\pi), \sqrt{2}(\cos \frac{19}{12}\pi + i \sin \frac{19}{12}\pi)$$