代幾 I 計算演習 [問題] (2007/05/24)

問. 次の二つの多項式の最大公約数を求めなさい。

Q.1

$$x^5 - x^4 - x^3 + 2x - 1, \quad x^3 + x^2 - 1$$

Q.2

$$x^5 - 4x^4 + 7x^3 - 9x^2 + 8x - 6, \quad x^4 - 3x^3 + 3x^2 - 4x + 3$$

Q.3

$$x^5 - 3x^4 + 4x^3 - x^2 - 2x + 2, \quad x^4 - 2x^3 + x^2 + x - 1$$

Q.4

$$x^4 - 2x^3 + 3x^2 - 2x + 2, \quad x^3 - x^2 + x - 1$$

Q.5

$$x^4 - x^3 - 2x^2 + 3x - 1, \quad x^3 - 3x^2 + 3x - 1$$

Q.6

$$x^4 + x^2, \quad x^3 - x^2 + x - 1$$

Q.7

$$x^7 - x^6 + 2x^5 - 3x^4 + x^3 - x^2 + 1, \quad x^2 + 1$$

Q.8

$$x^3 + x^2, \quad x^2 - 1$$

Q.9

$$x^7 - x^6 - 2x^5 + 5x^4 + x^3 - 7x^2 + 3, \quad x^5 - x^4 - x^3 + 3x^2 - 2$$

Q.10

$$x^5 + 2x^4 - 4x^2 - x + 2, \quad x^4 + 3x^3 + 2x^2 - 3x - 3$$

Q.11

$x^3 - 4x^2 + 6x - 3, \quad x^2 - 2x + 1$

Q.12

$x^7 - 2x^5 + x^4 + 4x^3 - 2x^2 - 4x, \quad x^6 - x^5 - x^4 + 2x^3 + x^2 - 2x - 2$

Q.13

$x^7 - x^6 + x^5 - 2x^4 + 3x^3 - 2x + 1, \quad x^6 - x^3 + 2x^2 + x - 1$

Q.14

$x^4 - 2x + 1, \quad x^3 - x^2$

Q.15

$x^5 - 2x^4 + x^3 + x^2 - 2x - 1, \quad x^4 - x^3 - x^2$

Q.16

$x^4 + 3x^3 + 4x^2 + 3x + 1, \quad x^2 + 2x + 1$

Q.17

$x^5 + 4x^4 + 4x^3 - 8x^2 - 17x - 7, \quad x^4 + 2x^3 - x^2 - 6x - 3$

Q.18

$x^5 + 4x^4 - 2x^3 - 26x^2 - 37x - 12, \quad x^4 + 2x^3 - 7x^2 - 13x - 3$

Q.19

$x^3 - x^2 + 2, \quad x^2 - 1$

Q.20

$x^3 - x^2, \quad x^2 - 2x + 1$

Q.21

$x^4 + x^2, \quad x^3 + x^2 + x + 1$

Q.22

$x^5 - x^3 + 2x^2 - 2, \quad x^3 + x^2 - x - 1$

Q.23

$x^4 + x^3 - x^2 - 2x - 2, \quad x^2 - 2$

代幾 I 計算演習 [解答] (2007/05/24)

A.1

[問題] $x^5 - x^4 - x^3 + 2x - 1, \quad x^3 + x^2 - 1$

[計算] $x^5 - x^4 - x^3 + 2x - 1 \;=\; (x^3 + x^2 - 1)(x^2 - 2x + 1) \;+\; 0$

[解答] $x^3 + x^2 - 1$

A.2

[問題] $x^5 - 4x^4 + 7x^3 - 9x^2 + 8x - 6, \quad x^4 - 3x^3 + 3x^2 - 4x + 3$

[計算]
$$\begin{array}{lcl} x^5 - 4x^4 + 7x^3 - 9x^2 + 8x - 6 & = & (x^4 - 3x^3 + 3x^2 - 4x + 3)(x - 1) \;+\; (x^3 - 2x^2 + x - 3) \\ x^4 - 3x^3 + 3x^2 - 4x + 3 & = & (x^3 - 2x^2 + x - 3)(x - 1) \;+\; 0 \end{array}$$

[解答] $x^3 - 2x^2 + x - 3$

A.3

[問題] $x^5 - 3x^4 + 4x^3 - x^2 - 2x + 2, \quad x^4 - 2x^3 + x^2 + x - 1$

[計算]
$$\begin{array}{lcl} x^5 - 3x^4 + 4x^3 - x^2 - 2x + 2 & = & (x^4 - 2x^3 + x^2 + x - 1)(x - 1) \;+\; (x^3 - x^2 + 1) \\ x^4 - 2x^3 + x^2 + x - 1 & = & (x^3 - x^2 + 1)(x - 1) \;+\; 0 \end{array}$$

[解答] $x^3 - x^2 + 1$

A.4

[問題] $x^4 - 2x^3 + 3x^2 - 2x + 2, \quad x^3 - x^2 + x - 1$

[計算]
$$\begin{array}{lcl} x^4 - 2x^3 + 3x^2 - 2x + 2 & = & (x^3 - x^2 + x - 1)(x - 1) \;+\; (x^2 + 1) \\ x^3 - x^2 + x - 1 & = & (x^2 + 1)(x - 1) \;+\; 0 \end{array}$$

[解答] $x^2 + 1$

A.5

[問題] $x^4 - x^3 - 2x^2 + 3x - 1, \quad x^3 - 3x^2 + 3x - 1$

[計算]
$$\begin{array}{lcl} x^4 - x^3 - 2x^2 + 3x - 1 & = & (x^3 - 3x^2 + 3x - 1)(x + 2) \;+\; (x^2 - 2x + 1) \\ x^3 - 3x^2 + 3x - 1 & = & (x^2 - 2x + 1)(x - 1) \;+\; 0 \end{array}$$

[解答] $x^2 - 2x + 1$

A.6

[問題] $x^4 + x^2, \quad x^3 - x^2 + x - 1$

[計算]
$$\begin{array}{lcll} x^4 + x^2 & = & (x^3 - x^2 + x - 1)(x + 1) & + \quad (x^2 + 1) \\ x^3 - x^2 + x - 1 & = & (x^2 + 1)(x - 1) & + \quad 0 \end{array}$$

[解答] $x^2 + 1$

A.7

[問題] $x^7 - x^6 + 2x^5 - 3x^4 + x^3 - x^2 + 1, \quad x^2 + 1$

[計算]
$$\begin{array}{lcl} x^7 - x^6 + 2x^5 - 3x^4 + x^3 - x^2 + 1 & = & (x^2 + 1)(x^5 - x^4 + x^3 - 2x^2 + 1) \\ & + & 0 \end{array}$$

[解答] $x^2 + 1$

A.8

[問題] $x^3 + x^2, \quad x^2 - 1$

[計算]
$$\begin{array}{lcll} x^3 + x^2 & = & (x^2 - 1)(x + 1) & + \quad (x + 1) \\ x^2 - 1 & = & (x + 1)(x - 1) & + \quad 0 \end{array}$$

[解答] $x + 1$

A.9

[問題] $x^7 - x^6 - 2x^5 + 5x^4 + x^3 - 7x^2 + 3, \quad x^5 - x^4 - x^3 + 3x^2 - 2$

[計算]
$$\begin{array}{lcl} x^7 - x^6 - 2x^5 + 5x^4 + x^3 - 7x^2 + 3 & = & (x^5 - x^4 - x^3 + 3x^2 - 2)(x^2 - 1) \\ & + & (x^4 - 2x^2 + 1) \\ x^5 - x^4 - x^3 + 3x^2 - 2 & = & (x^4 - 2x^2 + 1)(x - 1) \\ & + & (x^3 + x^2 - x - 1) \\ x^4 - 2x^2 + 1 & = & (x^3 + x^2 - x - 1)(x - 1) \\ & + & 0 \end{array}$$

[解答] $x^3 + x^2 - x - 1$

A.10

[問題] $x^5 + 2x^4 - 4x^2 - x + 2, \quad x^4 + 3x^3 + 2x^2 - 3x - 3$

[計算]
$$\begin{array}{lcll} x^5 + 2x^4 - 4x^2 - x + 2 & = & (x^4 + 3x^3 + 2x^2 - 3x - 3)(x - 1) & + \ (x^3 + x^2 - x - 1) \\ x^4 + 3x^3 + 2x^2 - 3x - 3 & = & (x^3 + x^2 - x - 1)(x + 2) & + \ (x^2 - 1) \\ x^3 + x^2 - x - 1 & = & (x^2 - 1)(x + 1) & + \ 0 \end{array}$$

[解答] $x^2 - 1$

A.11

[問題] $x^3 - 4x^2 + 6x - 3, \quad x^2 - 2x + 1$

[計算]
$$\begin{array}{lcll} x^3 - 4x^2 + 6x - 3 & = & (x^2 - 2x + 1)(x - 2) & + \ (x - 1) \\ x^2 - 2x + 1 & = & (x - 1)(x - 1) & + \ 0 \end{array}$$

[解答] $x - 1$

A.12

[問題] $x^7 - 2x^5 + x^4 + 4x^3 - 2x^2 - 4x, \quad x^6 - x^5 - x^4 + 2x^3 + x^2 - 2x - 2$

[計算]
$$\begin{array}{lcl} x^7 - 2x^5 + x^4 + 4x^3 - 2x^2 - 4x & = & (x^6 - x^5 - x^4 + 2x^3 + x^2 - 2x - 2)(x + 1) \\ & + & (x^3 - x^2 + 2) \\ x^6 - x^5 - x^4 + 2x^3 + x^2 - 2x - 2 & = & (x^3 - x^2 + 2)(x^3 - x - 1) \\ & + & 0 \end{array}$$

[解答] $x^3 - x^2 + 2$

A.13

[問題] $x^7 - x^6 + x^5 - 2x^4 + 3x^3 - 2x + 1, \quad x^6 - x^3 + 2x^2 + x - 1$

[計算]

$$\begin{array}{lcl}
x^7 - x^6 + x^5 - 2x^4 + 3x^3 - 2x + 1 & = & (x^6 - x^3 + 2x^2 + x - 1)(x - 1) \\
 & + & (x^5 - x^4 + x^2) \\
x^6 - x^3 + 2x^2 + x - 1 & = & (x^5 - x^4 + x^2)(x + 1) \\
 & + & (x^4 - 2x^3 + x^2 + x - 1) \\
x^5 - x^4 + x^2 & = & (x^4 - 2x^3 + x^2 + x - 1)(x + 1) \\
 & + & (x^3 - x^2 + 1) \\
x^4 - 2x^3 + x^2 + x - 1 & = & (x^3 - x^2 + 1)(x - 1) \\
 & + & 0
\end{array}$$

[解答] $x^3 - x^2 + 1$

A.14

[問題] $x^4 - 2x + 1, \quad x^3 - x^2$

[計算]

$$\begin{array}{lcll}
x^4 - 2x + 1 & = & (x^3 - x^2)(x + 1) & + \quad (x^2 - 2x + 1) \\
x^3 - x^2 & = & (x^2 - 2x + 1)(x + 1) & + \quad (x - 1) \\
x^2 - 2x + 1 & = & (x - 1)(x - 1) & + \quad 0
\end{array}$$

[解答] $x - 1$

A.15

[問題] $x^5 - 2x^4 + x^3 + x^2 - 2x - 1, \quad x^4 - x^3 - x^2$

[計算]

$$\begin{array}{lcll}
x^5 - 2x^4 + x^3 + x^2 - 2x - 1 & = & (x^4 - x^3 - x^2)(x - 1) & + \quad (x^3 - 2x - 1) \\
x^4 - x^3 - x^2 & = & (x^3 - 2x - 1)(x - 1) & + \quad (x^2 - x - 1) \\
x^3 - 2x - 1 & = & (x^2 - x - 1)(x + 1) & + \quad 0
\end{array}$$

[解答] $x^2 - x - 1$

A.16

[問題] $x^4 + 3x^3 + 4x^2 + 3x + 1, \quad x^2 + 2x + 1$

[計算] $x^4 + 3x^3 + 4x^2 + 3x + 1 = (x^2 + 2x + 1)(x^2 + x + 1) + 0$

[解答] $x^2 + 2x + 1$

A.17

[問題] $x^5+4x^4+4x^3-8x^2-17x-7, \quad x^4+2x^3-x^2-6x-3$

[計算]
$$\begin{array}{lcll} x^5+4x^4+4x^3-8x^2-17x-7 & = & (x^4+2x^3-x^2-6x-3)(x+2) & + \quad (x^3-2x-1) \\ x^4+2x^3-x^2-6x-3 & = & (x^3-2x-1)(x+2) & + \quad (x^2-x-1) \\ x^3-2x-1 & = & (x^2-x-1)(x+1) & + \quad 0 \end{array}$$

[解答] x^2-x-1

A.18

[問題] $x^5+4x^4-2x^3-26x^2-37x-12, \quad x^4+2x^3-7x^2-13x-3$

[計算]
$$\begin{array}{lcll} x^5+4x^4-2x^3-26x^2-37x-12 & = & (x^4+2x^3-7x^2-13x-3)(x+2) & + \quad (x^3+x^2-8x-6) \\ x^4+2x^3-7x^2-13x-3 & = & (x^3+x^2-8x-6)(x+1) & + \quad (x+3) \\ x^3+x^2-8x-6 & = & (x+3)(x^2-2x-2) & + \quad 0 \end{array}$$

[解答] $x+3$

A.19

[問題] $x^3-x^2+2, \quad x^2-1$

[計算]
$$\begin{array}{lcll} x^3-x^2+2 & = & (x^2-1)(x-1) & + \quad (x+1) \\ x^2-1 & = & (x+1)(x-1) & + \quad 0 \end{array}$$

[解答] $x+1$

A.20

[問題] $x^3-x^2, \quad x^2-2x+1$

[計算]
$$\begin{array}{lcll} x^3-x^2 & = & (x^2-2x+1)(x+1) & + \quad (x-1) \\ x^2-2x+1 & = & (x-1)(x-1) & + \quad 0 \end{array}$$

[解答] $x-1$

A.21

[問題] $x^4 + x^2, \quad x^3 + x^2 + x + 1$

[計算]
$$\begin{array}{lcll} x^4 + x^2 & = & (x^3 + x^2 + x + 1)(x - 1) & + \quad (x^2 + 1) \\ x^3 + x^2 + x + 1 & = & (x^2 + 1)(x + 1) & + \quad 0 \end{array}$$

[解答] $x^2 + 1$

A.22

[問題] $x^5 - x^3 + 2x^2 - 2, \quad x^3 + x^2 - x - 1$

[計算]
$$\begin{array}{lcll} x^5 - x^3 + 2x^2 - 2 & = & (x^3 + x^2 - x - 1)(x^2 - x + 1) & + \quad (x^2 - 1) \\ x^3 + x^2 - x - 1 & = & (x^2 - 1)(x + 1) & + \quad 0 \end{array}$$

[解答] $x^2 - 1$

A.23

[問題] $x^4 + x^3 - x^2 - 2x - 2, \quad x^2 - 2$

[計算] $x^4 + x^3 - x^2 - 2x - 2 \;=\; (x^2 - 2)(x^2 + x + 1) \;+\; 0$

[解答] $x^2 - 2$