

# 代幾 I 計算演習 [問題] (2007/12/20)

問. 次の行列の行列式を求めなさい

Q.1

$$\begin{vmatrix} 2 & 2 & 2 & -1 & -2 \\ -2 & -1 & -1 & 2 & 2 \\ -1 & 2 & -1 & -1 & 1 \\ 0 & -1 & -2 & -2 & 1 \\ 0 & -2 & 2 & -1 & 1 \end{vmatrix}$$

Q.2

$$\begin{vmatrix} 2 & -2 & 2 & 2 \\ -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & -2 \\ 2 & -1 & 2 & 2 \end{vmatrix}$$

Q.3

$$\begin{vmatrix} -2 & -1 & -1 & -1 & 0 \\ 1 & 0 & -1 & -2 & 2 \\ 1 & 1 & -2 & 2 & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 1 & -1 & -1 & 2 & -1 \end{vmatrix}$$

Q.4

$$\begin{vmatrix} -2 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ -2 & 1 & -2 & 0 \\ 0 & 2 & 2 & 2 \end{vmatrix}$$

Q.5

$$\begin{vmatrix} -2 & 0 & 1 & -1 \\ -1 & -1 & -1 & 2 \\ -2 & -1 & -2 & 2 \\ 2 & -2 & 2 & -2 \end{vmatrix}$$

Q.6

$$\begin{vmatrix} -1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 \\ -2 & 1 & -1 & -1 & 2 \\ 2 & -1 & -1 & 1 & 0 \\ 0 & 2 & -2 & 2 & -1 \end{vmatrix}$$

Q.7

$$\begin{vmatrix} 0 & 2 & 0 & 1 & 2 \\ 1 & -2 & -1 & 1 & -2 \\ 1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & -2 & 1 \\ 0 & 1 & -1 & 0 & -1 \end{vmatrix}$$

Q.8

$$\begin{vmatrix} 0 & -2 & 2 & -1 & 1 \\ 2 & 0 & -2 & 2 & -1 \\ 2 & -1 & 1 & -1 & -1 \\ 1 & 2 & 0 & -2 & -1 \\ -1 & 2 & -2 & 2 & 1 \end{vmatrix}$$

Q.9

$$\begin{vmatrix} 0 & 2 & 1 & 1 & -1 \\ -2 & 1 & -1 & -1 & 0 \\ 2 & -2 & 2 & -2 & -1 \\ -1 & -1 & -1 & 0 & -1 \\ 0 & 2 & 2 & 1 & 0 \end{vmatrix}$$

Q.10

$$\begin{vmatrix} 1 & 1 & -2 & 0 \\ 2 & -1 & 2 & 0 \\ 2 & -1 & -2 & -2 \\ 1 & -2 & 0 & -1 \end{vmatrix}$$

Q.11

$$\begin{vmatrix} 1 & -1 & 2 & 1 & -1 \\ 2 & 2 & 0 & 0 & 2 \\ 1 & -1 & -2 & 2 & -1 \\ -2 & 2 & -1 & -2 & -2 \\ -2 & -2 & 2 & 1 & 1 \end{vmatrix}$$

# 代幾 I 計算演習 [解答] (2007/12/20)

A.1

$$\begin{vmatrix} 2 & 2 & 2 & -1 & -2 \\ -2 & -1 & -1 & 2 & 2 \\ -1 & 2 & -1 & -1 & 1 \\ 0 & -1 & -2 & -2 & 1 \\ 0 & -2 & 2 & -1 & 1 \end{vmatrix} = 1 \begin{vmatrix} 2 & 2 & 2 & -1 & -2 \\ 0 & 1 & 1 & 1 & 0 \\ -1 & 2 & -1 & -1 & 1 \\ 0 & -1 & -2 & -2 & 1 \\ 0 & -2 & 2 & -1 & 1 \end{vmatrix}$$

左  $R(2,1;1)$ ; 2行目に  
1行目を1倍して、加  
える

$$= 1 \begin{vmatrix} 2 & 2 & 2 & -1 & -2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 3 & 0 & -\frac{3}{2} & 0 \\ 0 & -1 & -2 & -2 & 1 \\ 0 & -2 & 2 & -1 & 1 \end{vmatrix}$$

左  $R(3,1;\frac{1}{2})$ ; 3行目に  
1行目を $\frac{1}{2}$ 倍して、加  
える

$$= 2 \begin{vmatrix} 1 & 1 & 1 & 0 \\ 3 & 0 & -\frac{3}{2} & 0 \\ -1 & -2 & -2 & 1 \\ -2 & 2 & -1 & 1 \end{vmatrix}$$

左  $R(3,2;-3)$ ; 3行目  
に2行目を $-3$ 倍して、  
加える

$$= 2 \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & -\frac{9}{2} & 0 \\ -1 & -2 & -2 & 1 \\ -2 & 2 & -1 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & -\frac{9}{2} & 0 \\ 0 & -1 & -1 & 1 \\ -2 & 2 & -1 & 1 \end{vmatrix}$$

左  $R(4,2;1)$ ; 4行目に  
2行目を1倍して、加  
える

$$= 2 \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & -\frac{9}{2} & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 4 & 1 & 1 \end{vmatrix}$$

左  $R(5,2;2)$ ; 5行目に  
2行目を2倍して、加  
える

$$= 2 \begin{vmatrix} -3 & -\frac{9}{2} & 0 \\ -1 & -1 & 1 \\ 4 & 1 & 1 \\ -3 & -\frac{9}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 4 & 1 & 1 \end{vmatrix}$$

左  $R(4,3;-\frac{1}{3})$ ; 4行目  
に3行目を $-\frac{1}{3}$ 倍して、  
加える

$$= 2 \begin{vmatrix} -3 & -\frac{9}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & -5 & 1 \end{vmatrix}$$

左  $R(5,3;\frac{4}{3})$ ; 5行目に  
3行目を $\frac{4}{3}$ 倍して、加  
える

$$= -6 \begin{vmatrix} \frac{1}{2} & 1 \\ -5 & 1 \end{vmatrix}$$

$$= -6 \times (\frac{1}{2} \times 1 - 1 \times (-5))$$

$$= -33$$

A.2

$$\begin{aligned}
 \begin{vmatrix} 2 & -2 & 2 & 2 \\ -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & -2 \\ 2 & -1 & 2 & 2 \end{vmatrix} &= 1 \begin{vmatrix} 2 & -2 & 2 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 2 & -1 & 2 & 2 \end{vmatrix} && \text{左 } R(2,1;\frac{1}{2}) ; 2 \text{ 行目に} \\
 &&& \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\
 &&& \text{える} \\
 &= 1 \begin{vmatrix} 2 & -2 & 2 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & 0 & 0 \end{vmatrix} && \text{左 } R(4,1;-1) ; 4 \text{ 行目} \\
 &&& \text{に 1 行目を } -1 \text{ 倍して、} \\
 &&& \text{加える} \\
 &= 2 \begin{vmatrix} 0 & 0 & 2 \\ 1 & 0 & -2 \\ 1 & 0 & 0 \end{vmatrix} \\
 &= -2 \begin{vmatrix} 1 & 0 & -2 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{vmatrix} && \text{左 } P(3,2) ; 3 \text{ 行目と } 2 \text{ 行目を交換} \\
 &= -2 \begin{vmatrix} 1 & 0 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{vmatrix} && \text{左 } R(4,2;-1) ; 4 \text{ 行目} \\
 &&& \text{に } 2 \text{ 行目を } -1 \text{ 倍して、} \\
 &&& \text{加える} \\
 &= -2 \begin{vmatrix} 0 & 2 \\ 0 & 2 \end{vmatrix} \\
 &= -2 \times (0 \times 2 - 2 \times 0) \\
 &= 0
 \end{aligned}$$

## A.3

$$\begin{aligned}
& \left| \begin{array}{ccccc} -2 & -1 & -1 & -1 & 0 \\ 1 & 0 & -1 & -2 & 2 \\ 1 & 1 & -2 & 2 & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 1 & -1 & -1 & 2 & -1 \end{array} \right| = 1 \left| \begin{array}{ccccc} -2 & -1 & -1 & -1 & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 2 \\ 1 & 1 & -2 & 2 & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 1 & -1 & -1 & 2 & -1 \end{array} \right| & \text{左 } R(2,1;\frac{1}{2}); 2 \text{ 行目に} \\
& & & & & \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\
& & & & & \text{える} \\
& = 1 \left| \begin{array}{ccccc} -2 & -1 & -1 & -1 & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 2 \\ 0 & \frac{1}{2} & -\frac{5}{2} & \frac{3}{2} & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 1 & -1 & -1 & 2 & -1 \end{array} \right| & \text{左 } R(3,1;\frac{1}{2}); 3 \text{ 行目に} \\
& & & & & \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\
& & & & & \text{える} \\
& = 1 \left| \begin{array}{ccccc} -2 & -1 & -1 & -1 & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 2 \\ 0 & \frac{1}{2} & -\frac{5}{2} & \frac{3}{2} & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 0 & -\frac{3}{2} & -\frac{3}{2} & \frac{3}{2} & -1 \end{array} \right| & \text{左 } R(5,1;\frac{1}{2}); 5 \text{ 行目に} \\
& & & & & \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\
& & & & & \text{える} \\
& = -2 \left| \begin{array}{ccccc} -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 2 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 2 \\ -1 & 0 & -2 & 0 \\ -\frac{3}{2} & -\frac{3}{2} & \frac{3}{2} & -1 \\ -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 2 \\ 0 & -4 & -1 & 3 \\ -1 & 0 & -2 & 0 \\ -\frac{3}{2} & -\frac{3}{2} & \frac{3}{2} & -1 \end{array} \right| & \text{左 } R(3,2;1); 3 \text{ 行目に} \\
& & & & & \text{2 行目を 1 倍して、加} \\
& & & & & \text{える} \\
& = -2 \left| \begin{array}{ccccc} -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 2 \\ 0 & -4 & -1 & 3 \\ 0 & 3 & 3 & -4 \\ -\frac{3}{2} & -\frac{3}{2} & \frac{3}{2} & -1 \end{array} \right| & \text{左 } R(4,2;-2); 4 \text{ 行目に} \\
& & & & & \text{2 行目を } -2 \text{ 倍して、} \\
& & & & & \text{加える} \\
& = -2 \left| \begin{array}{ccccc} -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 2 \\ 0 & -4 & -1 & 3 \\ 0 & 3 & 3 & -4 \\ 0 & 3 & 9 & -7 \end{array} \right| & \text{左 } R(5,2;-3); 5 \text{ 行目に} \\
& & & & & \text{2 行目を } -3 \text{ 倍して、} \\
& & & & & \text{加える} \\
& = 1 \left| \begin{array}{ccc} -4 & -1 & 3 \\ 3 & 3 & -4 \\ 3 & 9 & -7 \end{array} \right| & \\
& = 1 \left| \begin{array}{ccc} -4 & -1 & 3 \\ 0 & \frac{9}{4} & -\frac{7}{4} \\ 3 & 9 & -7 \end{array} \right| & \text{左 } R(4,3;\frac{3}{4}); 4 \text{ 行目に} \\
& & & & & \text{3 行目を } \frac{3}{4} \text{ 倍して、加} \\
& & & & & \text{える} \\
& = 1 \left| \begin{array}{ccc} -4 & -1 & 3 \\ 0 & \frac{9}{4} & -\frac{7}{4} \\ 0 & \frac{33}{4} & -\frac{19}{4} \end{array} \right| & \text{左 } R(5,3;\frac{3}{4}); 5 \text{ 行目に} \\
& & & & & \text{3 行目を } \frac{3}{4} \text{ 倍して、加} \\
& & & & & \text{える} \\
& = -4 \left| \begin{array}{cc} \frac{9}{4} & -\frac{7}{4} \\ \frac{33}{4} & -\frac{19}{4} \end{array} \right| & \\
& = -4 \times \left( \frac{9}{4} \times \frac{1}{4} \left( -\frac{19}{4} \right) - \left( -\frac{7}{4} \right) \times \frac{33}{4} \right) & \\
& = -15 &
\end{aligned}$$

## A.4

$$\begin{aligned}
 \begin{vmatrix} -2 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ -2 & 1 & -2 & 0 \\ 0 & 2 & 2 & 2 \end{vmatrix} &= 1 \begin{vmatrix} -2 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & -3 & -2 \\ 0 & 2 & 2 & 2 \end{vmatrix} && \text{左 } R(3,1;-1) ; 3 \text{ 行目} \\
 &&& \text{に 1 行目を } -1 \text{ 倍して、} \\
 &&& \text{加える} \\
 &= -2 \begin{vmatrix} 2 & 0 & 2 \\ 0 & -3 & -2 \\ 2 & 2 & 2 \end{vmatrix} \\
 &= -2 \begin{vmatrix} 2 & 0 & 2 \\ 0 & -3 & -2 \\ 0 & 2 & 0 \end{vmatrix} && \text{左 } R(4,2;-1) ; 4 \text{ 行目} \\
 &&& \text{に 2 行目を } -1 \text{ 倍して、} \\
 &&& \text{加える} \\
 &= -4 \begin{vmatrix} -3 & -2 \\ 2 & 0 \end{vmatrix} \\
 &= -4 \times ((-3) \times 0 - (-2) \times 2) \\
 &= -16
 \end{aligned}$$

## A.5

$$\begin{aligned}
& \left| \begin{array}{cccc} -2 & 0 & 1 & -1 \\ -1 & -1 & -1 & 2 \\ -2 & -1 & -2 & 2 \\ 2 & -2 & 2 & -2 \end{array} \right| = 1 \left| \begin{array}{cccc} -2 & 0 & 1 & -1 \\ 0 & -1 & -\frac{3}{2} & \frac{5}{2} \\ -2 & -1 & -2 & 2 \\ 2 & -2 & 2 & -2 \end{array} \right| & \text{左 } R(2,1;-\frac{1}{2}) ; 2 \text{ 行目} \\
& & & \text{に 1 行目を } -\frac{1}{2} \text{ 倍して、} \\
& & & \text{加える} \\
& = 1 \left| \begin{array}{cccc} -2 & 0 & 1 & -1 \\ 0 & -1 & -\frac{3}{2} & \frac{5}{2} \\ 0 & -1 & -3 & 3 \\ 2 & -2 & 2 & -2 \end{array} \right| & \text{左 } R(3,1;-1) ; 3 \text{ 行目} \\
& & & \text{に 1 行目を } -1 \text{ 倍して、} \\
& & & \text{加える} \\
& = 1 \left| \begin{array}{cccc} -2 & 0 & 1 & -1 \\ 0 & -1 & -\frac{3}{2} & \frac{5}{2} \\ 0 & -1 & -3 & 3 \\ 0 & -2 & 3 & -3 \end{array} \right| & \text{左 } R(4,1;1) ; 4 \text{ 行目に} \\
& & & \text{1 行目を 1 倍して、加} \\
& & & \text{える} \\
& = -2 \left| \begin{array}{ccc} -1 & -\frac{3}{2} & \frac{5}{2} \\ -1 & -3 & 3 \\ -2 & 3 & -3 \end{array} \right| \\
& = -2 \left| \begin{array}{ccc} -1 & -\frac{3}{2} & \frac{5}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} \\ -2 & 3 & -3 \end{array} \right| & \text{左 } R(3,2;-1) ; 3 \text{ 行目} \\
& & & \text{に 2 行目を } -1 \text{ 倍して、} \\
& & & \text{加える} \\
& = -2 \left| \begin{array}{ccc} -1 & -\frac{3}{2} & \frac{5}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 6 & -8 \end{array} \right| & \text{左 } R(4,2;-2) ; 4 \text{ 行目} \\
& & & \text{に 2 行目を } -2 \text{ 倍して、} \\
& & & \text{加える} \\
& = 2 \left| \begin{array}{cc} -\frac{3}{2} & \frac{1}{2} \\ 6 & -8 \end{array} \right| \\
& = 2 \times \left( (-\frac{3}{2}) \times (-8) - \frac{1}{2} \times 6 \right) \\
& = 18
\end{aligned}$$



A.7

$$\begin{aligned}
 & \left| \begin{array}{ccccc} 0 & 2 & 0 & 1 & 2 \\ 1 & -2 & -1 & 1 & -2 \\ 1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & -2 & 1 \\ 0 & 1 & -1 & 0 & -1 \end{array} \right| = -1 \left| \begin{array}{ccccc} 1 & -2 & -1 & 1 & -2 \\ 0 & 2 & 0 & 1 & 2 \\ 1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & -2 & 1 \\ 0 & 1 & -1 & 0 & -1 \end{array} \right| & \text{左 } P(2,1) ; 2 \text{ 行目と } 1 \text{ 行目を交換} \\
 & = -1 \left| \begin{array}{ccccc} 1 & -2 & -1 & 1 & -2 \\ 0 & 2 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ -1 & 1 & -1 & -2 & 1 \\ 0 & 1 & -1 & 0 & -1 \end{array} \right| & \text{左 } R(3,1;-1) ; 3 \text{ 行目} \\
 & & & & \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\
 & & & & \text{加える} \\
 & = -1 \left| \begin{array}{ccccc} 1 & -2 & -1 & 1 & -2 \\ 0 & 2 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & -1 & -2 & -1 & -1 \\ 0 & 1 & -1 & 0 & -1 \end{array} \right| & \text{左 } R(4,1;1) ; 4 \text{ 行目に} \\
 & & & & \text{ } 1 \text{ 行目を } 1 \text{ 倍して、加} \\
 & & & & \text{える} \\
 & = -1 \left| \begin{array}{ccccc} 2 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ -1 & -2 & -1 & -1 \\ 1 & -1 & 0 & -1 \\ 2 & 0 & 1 & 2 \\ 0 & 0 & -\frac{1}{2} & 0 \\ -1 & -2 & -1 & -1 \\ 1 & -1 & 0 & -1 \end{array} \right| & \text{左 } R(3,2;-\frac{1}{2}) ; 3 \text{ 行目} \\
 & & & & \text{に } 2 \text{ 行目を } -\frac{1}{2} \text{ 倍して、} \\
 & & & & \text{加える} \\
 & = -1 \left| \begin{array}{ccccc} 2 & 0 & 1 & 2 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & -2 & -\frac{1}{2} & 0 \\ 1 & -1 & 0 & -1 \end{array} \right| & \text{左 } R(4,2;\frac{1}{2}) ; 4 \text{ 行目に} \\
 & & & & \text{ } 2 \text{ 行目を } \frac{1}{2} \text{ 倍して、加} \\
 & & & & \text{える} \\
 & = -1 \left| \begin{array}{ccccc} 2 & 0 & 1 & 2 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & -2 & -\frac{1}{2} & 0 \\ 0 & -1 & -\frac{1}{2} & -2 \end{array} \right| & \text{左 } R(5,2;-\frac{1}{2}) ; 5 \text{ 行目} \\
 & & & & \text{に } 2 \text{ 行目を } -\frac{1}{2} \text{ 倍して、} \\
 & & & & \text{加える} \\
 & = -2 \left| \begin{array}{ccc} 0 & -\frac{1}{2} & 0 \\ -2 & -\frac{1}{2} & 0 \\ -1 & -\frac{1}{2} & -2 \end{array} \right| \\
 & = 2 \left| \begin{array}{ccc} -2 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 \\ -1 & -\frac{1}{2} & -2 \end{array} \right| & \text{左 } P(4,3) ; 4 \text{ 行目と } 3 \text{ 行目を交換} \\
 & = 2 \left| \begin{array}{ccc} -2 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{4} & -2 \end{array} \right| & \text{左 } R(5,3;-\frac{1}{2}) ; 5 \text{ 行目} \\
 & & & & \text{に } 3 \text{ 行目を } -\frac{1}{2} \text{ 倍して、} \\
 & & & & \text{加える} \\
 & = -4 \left| \begin{array}{cc} -\frac{1}{2} & 0 \\ -\frac{1}{4} & -2 \end{array} \right| \\
 & = -4 \times ((-\frac{1}{2}) \times (-2) - 0 \times (-\frac{1}{4})) \\
 & = -4
 \end{aligned}$$



$$\begin{aligned}
& \left| \begin{array}{ccccc} 0 & -2 & 2 & -1 & 1 \\ 2 & 0 & -2 & 2 & -1 \\ 2 & -1 & 1 & -1 & -1 \\ 1 & 2 & 0 & -2 & -1 \\ -1 & 2 & -2 & 2 & 1 \end{array} \right| = -1 \left| \begin{array}{ccccc} 2 & 0 & -2 & 2 & -1 \\ 0 & -2 & 2 & -1 & 1 \\ 2 & -1 & 1 & -1 & -1 \\ 1 & 2 & 0 & -2 & -1 \\ -1 & 2 & -2 & 2 & 1 \end{array} \right| & \text{左 P(2,1); 2行目と1行目を交換} \\
& = -1 \left| \begin{array}{ccccc} 2 & 0 & -2 & 2 & -1 \\ 0 & -2 & 2 & -1 & 1 \\ 0 & -1 & 3 & -3 & 0 \\ 1 & 2 & 0 & -2 & -1 \\ -1 & 2 & -2 & 2 & 1 \end{array} \right| & \text{左 R(3,1;-1); 3行目に1行目を-1倍して、加える} \\
& = -1 \left| \begin{array}{ccccc} 2 & 0 & -2 & 2 & -1 \\ 0 & -2 & 2 & -1 & 1 \\ 0 & -1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & -\frac{1}{2} \\ -1 & 2 & -2 & 2 & 1 \end{array} \right| & \text{左 R(4,1;-\frac{1}{2}); 4行目に1行目を-\frac{1}{2}倍して、加える} \\
& = -1 \left| \begin{array}{ccccc} 2 & 0 & -2 & 2 & -1 \\ 0 & -2 & 2 & -1 & 1 \\ 0 & -1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & -\frac{1}{2} \\ 0 & 2 & -3 & 3 & \frac{1}{2} \end{array} \right| & \text{左 R(5,1;\frac{1}{2}); 5行目に1行目を\frac{1}{2}倍して、加える} \\
& = -2 \left| \begin{array}{ccccc} -2 & 2 & -1 & 1 \\ -1 & 3 & -3 & 0 \\ 2 & 1 & -3 & -\frac{1}{2} \\ 2 & -3 & 3 & \frac{1}{2} \end{array} \right| \\
& = -2 \left| \begin{array}{ccccc} -2 & 2 & -1 & 1 \\ 0 & 2 & -\frac{5}{2} & -\frac{1}{2} \\ 2 & 1 & -3 & -\frac{1}{2} \\ 2 & -3 & 3 & \frac{1}{2} \end{array} \right| & \text{左 R(3,2;-\frac{1}{2}); 3行目に2行目を-\frac{1}{2}倍して、加える} \\
& = -2 \left| \begin{array}{ccccc} -2 & 2 & -1 & 1 \\ 0 & 2 & -\frac{5}{2} & -\frac{1}{2} \\ 0 & 3 & -4 & \frac{1}{2} \\ 2 & -3 & 3 & \frac{1}{2} \end{array} \right| & \text{左 R(4,2;1); 4行目に2行目を1倍して、加える} \\
& = -2 \left| \begin{array}{ccccc} -2 & 2 & -1 & 1 \\ 0 & 2 & -\frac{5}{2} & -\frac{1}{2} \\ 0 & 3 & -4 & \frac{1}{2} \\ 0 & -1 & 2 & \frac{3}{2} \end{array} \right| & \text{左 R(5,2;1); 5行目に2行目を1倍して、加える} \\
& = 4 \left| \begin{array}{ccc} 2 & -\frac{5}{2} & -\frac{1}{2} \\ 3 & -4 & \frac{3}{2} \\ -1 & 2 & \frac{3}{2} \end{array} \right| \\
& = 4 \left| \begin{array}{ccc} 2 & -\frac{5}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{4} & \frac{5}{4} \\ -1 & 2 & \frac{3}{2} \end{array} \right| & \text{左 R(4,3;-\frac{3}{2}); 4行目に3行目を-\frac{3}{2}倍して、加える} \\
& = 4 \left| \begin{array}{ccc} 2 & -\frac{5}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{4} & \frac{5}{4} \\ 0 & \frac{3}{4} & \frac{5}{4} \end{array} \right| & \text{左 R(5,3;\frac{1}{2}); 5行目に3行目を\frac{1}{2}倍して、加える} \\
& = 8 \left| \begin{array}{cc} -\frac{1}{4} & \frac{5}{4} \\ \frac{3}{4} & \frac{5}{4} \end{array} \right| \\
& = 8 \times \left( (-\frac{1}{4}) \times \frac{5}{4} - \frac{5}{4} \times \frac{3}{4} \right) \\
& = -10
\end{aligned}$$

A.9

$$\begin{aligned}
 & \left| \begin{array}{ccccc} 0 & 2 & 1 & 1 & -1 \\ -2 & 1 & -1 & -1 & 0 \\ 2 & -2 & 2 & -2 & -1 \\ -1 & -1 & -1 & 0 & -1 \\ 0 & 2 & 2 & 1 & 0 \end{array} \right| = -1 \left| \begin{array}{ccccc} -2 & 1 & -1 & -1 & 0 \\ 0 & 2 & 1 & 1 & -1 \\ 2 & -2 & 2 & -2 & -1 \\ -1 & -1 & -1 & 0 & -1 \\ 0 & 2 & 2 & 1 & 0 \end{array} \right| & \text{左 } P(2,1) ; 2 \text{ 行目と } 1 \text{ 行目を交換} \\
 & = -1 \left| \begin{array}{ccccc} -2 & 1 & -1 & -1 & 0 \\ 0 & 2 & 1 & 1 & -1 \\ 0 & -1 & 1 & -3 & -1 \\ -1 & -1 & -1 & 0 & -1 \\ 0 & 2 & 2 & 1 & 0 \end{array} \right| & \text{左 } R(3,1;1) ; 3 \text{ 行目に} \\
 & & & & \text{1 行目を } 1 \text{ 倍して、加} \\
 & & & & \text{える} \\
 & = -1 \left| \begin{array}{ccccc} -2 & 1 & -1 & -1 & 0 \\ 0 & 2 & 1 & 1 & -1 \\ 0 & -1 & 1 & -3 & -1 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & -1 \\ 0 & 2 & 2 & 1 & 0 \end{array} \right| & \text{左 } R(4,1;-\frac{1}{2}) ; 4 \text{ 行目} \\
 & & & & \text{に } 1 \text{ 行目を } -\frac{1}{2} \text{ 倍して、} \\
 & & & & \text{加える} \\
 & = 2 \left| \begin{array}{ccccc} 2 & 1 & 1 & -1 & \\ -1 & 1 & -3 & -1 & \\ -\frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & -1 & \\ 2 & 2 & 1 & 0 & \end{array} \right| \\
 & = 2 \left| \begin{array}{ccccc} 2 & 1 & 1 & -1 & \\ 0 & \frac{3}{2} & -\frac{5}{2} & -\frac{3}{2} & \\ -\frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & -1 & \\ 2 & 2 & 1 & 0 & \end{array} \right| & \text{左 } R(3,2;\frac{1}{2}) ; 3 \text{ 行目に} \\
 & & & & \text{2 行目を } \frac{1}{2} \text{ 倍して、加} \\
 & & & & \text{える} \\
 & = 2 \left| \begin{array}{ccccc} 2 & 1 & 1 & -1 & \\ 0 & \frac{3}{2} & -\frac{5}{2} & -\frac{3}{2} & \\ 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \\ 2 & 2 & 1 & 0 & \end{array} \right| & \text{左 } R(4,2;\frac{3}{4}) ; 4 \text{ 行目に} \\
 & & & & \text{2 行目を } \frac{3}{4} \text{ 倍して、加} \\
 & & & & \text{える} \\
 & = 2 \left| \begin{array}{ccccc} 2 & 1 & 1 & -1 & \\ 0 & \frac{3}{2} & -\frac{5}{2} & -\frac{3}{2} & \\ 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \\ 0 & 1 & 0 & 1 & \end{array} \right| & \text{左 } R(5,2;-1) ; 5 \text{ 行目} \\
 & & & & \text{に } 2 \text{ 行目を } -1 \text{ 倍して、} \\
 & & & & \text{加える} \\
 & = 4 \left| \begin{array}{ccc} \frac{3}{2} & -\frac{5}{2} & -\frac{3}{2} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 1 & 0 & 1 \end{array} \right| \\
 & = 4 \left| \begin{array}{ccc} \frac{3}{2} & -\frac{5}{2} & -\frac{3}{2} \\ 0 & \frac{1}{4} & -\frac{1}{4} \\ 1 & 0 & 1 \end{array} \right| & \text{左 } R(4,3;-\frac{1}{6}) ; 4 \text{ 行目} \\
 & & & & \text{に } 3 \text{ 行目を } -\frac{1}{6} \text{ 倍して、} \\
 & & & & \text{加える} \\
 & = 4 \left| \begin{array}{ccc} \frac{3}{2} & -\frac{5}{2} & -\frac{3}{2} \\ 0 & \frac{1}{4} & -\frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{2} \end{array} \right| & \text{左 } R(5,3;-\frac{2}{3}) ; 5 \text{ 行目} \\
 & & & & \text{に } 3 \text{ 行目を } -\frac{2}{3} \text{ 倍して、} \\
 & & & & \text{加える} \\
 & = 6 \left| \begin{array}{cc} \frac{5}{3} & -\frac{3}{2} \\ \frac{1}{3} & 2 \end{array} \right| \\
 & = 6 \times (\frac{5}{3} \times 2 - (-\frac{3}{2}) \times \frac{1}{3}) \\
 & = 35
 \end{aligned}$$

A.10

$$\begin{aligned}
 \begin{vmatrix} 1 & 1 & -2 & 0 \\ 2 & -1 & 2 & 0 \\ 2 & -1 & -2 & -2 \\ 1 & -2 & 0 & -1 \end{vmatrix} &= 1 \begin{vmatrix} 1 & 1 & -2 & 0 \\ 0 & -3 & 6 & 0 \\ 2 & -1 & -2 & -2 \\ 1 & -2 & 0 & -1 \end{vmatrix} && \text{左 R(2,1;-2) ; 2 行目} \\
 & & & \text{に 1 行目を } -2 \text{ 倍して、} \\
 & & & \text{加える} \\
 &= 1 \begin{vmatrix} 1 & 1 & -2 & 0 \\ 0 & -3 & 6 & 0 \\ 0 & -3 & 2 & -2 \\ 1 & -2 & 0 & -1 \end{vmatrix} && \text{左 R(3,1;-2) ; 3 行目} \\
 & & & \text{に 1 行目を } -2 \text{ 倍して、} \\
 & & & \text{加える} \\
 &= 1 \begin{vmatrix} 1 & 1 & -2 & 0 \\ 0 & -3 & 6 & 0 \\ 0 & -3 & 2 & -2 \\ 0 & -3 & 2 & -1 \end{vmatrix} && \text{左 R(4,1;-1) ; 4 行目} \\
 & & & \text{に 1 行目を } -1 \text{ 倍して、} \\
 & & & \text{加える} \\
 &= 1 \begin{vmatrix} -3 & 6 & 0 \\ -3 & 2 & -2 \\ -3 & 2 & -1 \end{vmatrix} && \\
 &= 1 \begin{vmatrix} -3 & 6 & 0 \\ 0 & -4 & -2 \\ -3 & 2 & -1 \end{vmatrix} && \text{左 R(3,2;-1) ; 3 行目} \\
 & & & \text{に 2 行目を } -1 \text{ 倍して、} \\
 & & & \text{加える} \\
 &= 1 \begin{vmatrix} -3 & 6 & 0 \\ 0 & -4 & -2 \\ 0 & -4 & -1 \end{vmatrix} && \text{左 R(4,2;-1) ; 4 行目} \\
 & & & \text{に 2 行目を } -1 \text{ 倍して、} \\
 & & & \text{加える} \\
 &= -3 \begin{vmatrix} -4 & -2 \\ -4 & -1 \end{vmatrix} && \\
 &= -3 \times ((-4) \times (-1) - (-2) \times (-4)) && \\
 &= 12 && 
 \end{aligned}$$

A.11

$$\begin{aligned}
 & \left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & -1 \\ 2 & 2 & 0 & 0 & 2 \\ 1 & -1 & -2 & 2 & -1 \\ -2 & 2 & -1 & -2 & -2 \\ -2 & -2 & 2 & 1 & 1 \end{array} \right| = 1 \left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & -1 \\ 0 & 4 & -4 & -2 & 4 \\ 1 & -1 & -2 & 2 & -1 \\ -2 & 2 & -1 & -2 & -2 \\ -2 & -2 & 2 & 1 & 1 \end{array} \right| \quad \begin{array}{l} \text{左 } R(2,1;-2) ; 2 \text{ 行目} \\ \text{に } 1 \text{ 行目を } -2 \text{ 倍して、} \\ \text{加える} \end{array} \\
 & = 1 \left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & -1 \\ 0 & 4 & -4 & -2 & 4 \\ 0 & 0 & -4 & 1 & 0 \\ -2 & 2 & -1 & -2 & -2 \\ -2 & -2 & 2 & 1 & 1 \end{array} \right| \quad \begin{array}{l} \text{左 } R(3,1;-1) ; 3 \text{ 行目} \\ \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\ \text{加える} \end{array} \\
 & = 1 \left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & -1 \\ 0 & 4 & -4 & -2 & 4 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 3 & 0 & -4 \\ -2 & -2 & 2 & 1 & 1 \end{array} \right| \quad \begin{array}{l} \text{左 } R(4,1;2) ; 4 \text{ 行目に} \\ \text{ } 1 \text{ 行目を } 2 \text{ 倍して、加} \\ \text{える} \end{array} \\
 & = 1 \left| \begin{array}{ccccc} 1 & -1 & 2 & 1 & -1 \\ 0 & 4 & -4 & -2 & 4 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 3 & 0 & -4 \\ 0 & -4 & 6 & 3 & -1 \end{array} \right| \quad \begin{array}{l} \text{左 } R(5,1;2) ; 5 \text{ 行目に} \\ \text{ } 1 \text{ 行目を } 2 \text{ 倍して、加} \\ \text{える} \end{array} \\
 & = 1 \left| \begin{array}{cccc} 4 & -4 & -2 & 4 \\ 0 & -4 & 1 & 0 \\ 0 & 3 & 0 & -4 \\ -4 & 6 & 3 & -1 \end{array} \right| \\
 & = 1 \left| \begin{array}{cccc} 4 & -4 & -2 & 4 \\ 0 & -4 & 1 & 0 \\ 0 & 3 & 0 & -4 \\ 0 & 2 & 1 & 3 \end{array} \right| \quad \begin{array}{l} \text{左 } R(5,2;1) ; 5 \text{ 行目に} \\ \text{ } 2 \text{ 行目を } 1 \text{ 倍して、加} \\ \text{える} \end{array} \\
 & = 4 \left| \begin{array}{ccc} -4 & 1 & 0 \\ 3 & 0 & -4 \\ 2 & 1 & 3 \end{array} \right| \\
 & = 4 \left| \begin{array}{ccc} -4 & 1 & 0 \\ 0 & \frac{3}{4} & -4 \\ 2 & 1 & 3 \end{array} \right| \quad \begin{array}{l} \text{左 } R(4,3;\frac{3}{4}) ; 4 \text{ 行目に} \\ \text{ } 3 \text{ 行目を } \frac{3}{4} \text{ 倍して、加} \\ \text{える} \end{array} \\
 & = 4 \left| \begin{array}{ccc} -4 & 1 & 0 \\ 0 & \frac{3}{4} & -4 \\ 0 & \frac{3}{2} & 3 \end{array} \right| \quad \begin{array}{l} \text{左 } R(5,3;\frac{1}{2}) ; 5 \text{ 行目に} \\ \text{ } 3 \text{ 行目を } \frac{1}{2} \text{ 倍して、加} \\ \text{える} \end{array} \\
 & = -16 \left| \begin{array}{cc} \frac{3}{4} & -4 \\ \frac{3}{2} & 3 \end{array} \right| \\
 & = -16 \times \left( \frac{3}{4} \times 3 - (-4) \times \frac{3}{2} \right) \\
 & = -132
 \end{aligned}$$