代幾 I 計算演習 [問題] (2008/11/06)

問. 次の連立方程式をクラーメルの公式を用いて解きなさい。

Q.1

$$\begin{cases} x_0 + 11x_1 - 18x_2 = -54 \\ x_0 - 8x_1 + 12x_2 = 33 \\ -x_0 - 4x_1 + 7x_2 = 22 \end{cases}$$

Q.2

$$\begin{cases} -5x_1 - 2x_2 + 4x_3 = 16 \\ x_0 + x_1 + 2x_2 - 2x_3 = -4 \\ x_0 - 5x_1 - 3x_2 + 3x_3 = 8 \\ -2x_0 + 8x_1 + x_2 - 4x_3 = -21 \end{cases}$$

Q.3

$$\begin{cases} -x_0 + 3x_1 - 3x_2 = -6 \\ -3x_0 - 18x_1 + 10x_2 + 8x_3 = 55 \\ 2x_0 + 11x_1 - 6x_2 - 5x_3 = -34 \\ x_2 = -1 \end{cases}$$

Q.4

$$\begin{cases} x_0 - x_1 - 2x_2 = -7 \\ -x_0 + 4x_1 + 3x_2 - 3x_3 = 10 \\ -x_0 - x_2 + 2x_3 = -3 \\ x_0 - x_1 + x_2 - x_3 = 3 \end{cases}$$

Q.5

$$\begin{cases} x_0 - 3x_1 + x_2 = 4 \\ -x_0 + 4x_1 - 2x_2 = -8 \\ -3x_0 + 4x_1 + 3x_2 = 10 \end{cases}$$

Q.6

$$\begin{cases} 8x_0 - 11x_1 = 36 \\ -13x_0 + 18x_1 = -59 \\ -7x_0 + 9x_1 + x_2 = -32 \end{cases}$$

Q.7

$$\begin{cases} x_0 = -4 \\ -3x_0 - 21x_1 + 13x_2 = 7 \\ 4x_0 + 34x_1 - 21x_2 = -8 \end{cases}$$

Q.8

$$\begin{cases} 2x_0 - 2x_1 + 2x_2 + x_3 = 1 \\ -3x_0 + 2x_1 - 3x_2 + x_3 = 8 \\ 2x_0 - x_1 + x_2 - x_3 = -8 \\ -2x_0 + 3x_1 - 4x_3 = -10 \end{cases}$$

Q.9

$$\begin{cases} -5x_0 - 3x_1 + 2x_2 + 5x_3 = 13 \\ -2x_0 - x_1 + x_2 = 0 \\ 4x_0 + 3x_1 - 2x_2 - 2x_3 = -3 \\ 2x_0 - x_1 - 3x_3 = -14 \end{cases}$$

Q.10

$$\begin{cases} -3x_0 + 5x_1 - 5x_2 = -21 \\ 9x_0 - 10x_1 + 8x_2 = 27 \\ -2x_0 + x_1 = 3 \end{cases}$$

Q.11

$$\begin{cases} 2x_1 - x_2 = -7 \\ -3x_0 + 4x_1 - x_2 = -22 \\ 5x_0 - 7x_1 + 2x_2 = 38 \end{cases}$$

Q.12

$$\begin{cases} -x_0 - x_1 + x_2 = 5 \\ 3x_0 - 2x_1 - x_2 = -6 \\ -5x_0 + 8x_1 = 2 \end{cases}$$

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A.1

$$|A| = \begin{vmatrix} 1 & 11 & -18 \\ 1 & -8 & 12 \\ -1 & -4 & 7 \end{vmatrix} = -1$$

$$|A_0| = \begin{vmatrix} -54 & 11 & -18 \\ 33 & -8 & 12 \\ 22 & -4 & 7 \end{vmatrix} = 3$$

$$|A_1| = \begin{vmatrix} 1 & -54 & -18 \\ 1 & 33 & 12 \\ -1 & 22 & 7 \end{vmatrix} = 3$$

$$|A_2| = \begin{vmatrix} 1 & 11 & -54 \\ 1 & -8 & 33 \\ -1 & -4 & 22 \end{vmatrix} = -1$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{3}{-1} = -3 \\ x_1 = \frac{|A_1|}{|A|} = \frac{3}{-1} = -3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-1}{-1} = 1 \end{cases}$$

A.2

$$|A| = \begin{vmatrix} 0 & -5 & -2 & 4 \\ 1 & 1 & 2 & -2 \\ 1 & -5 & -3 & 3 \\ -2 & 8 & 1 & -4 \end{vmatrix} = -1$$

$$|A_0| = \begin{vmatrix} 16 & -5 & -2 & 4 \\ -4 & 1 & 2 & -2 \\ 8 & -5 & -3 & 3 \\ -21 & 8 & 1 & -4 \end{vmatrix} = 2$$

$$|A_1| = \begin{vmatrix} 0 & 16 & -2 & 4 \\ 1 & -4 & 2 & -2 \\ 1 & 8 & -3 & 3 \\ -2 & -21 & 1 & -4 \end{vmatrix} = 2$$

$$|A_2| = \begin{vmatrix} 0 & -5 & 16 & 4 \\ 1 & 1 & -4 & -2 \\ 1 & -5 & 8 & 3 \\ -2 & 8 & -21 & -4 \end{vmatrix} = -3$$

$$|A_3| = \begin{vmatrix} 0 & -5 & -2 & 16 \\ 1 & 1 & 2 & -4 \\ 1 & -5 & -3 & 8 \\ -2 & 8 & 1 & -21 \end{vmatrix} = -3$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{2}{-1} = -2 \\ x_1 = \frac{|A_1|}{|A|} = \frac{2}{-1} = -2 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-3}{-1} = 3 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-3}{-1} = 3 \end{cases}$$

A.3

$$|A| = \begin{vmatrix} -1 & 3 & -3 & 0 \\ -3 & -18 & 10 & 8 \\ 2 & 11 & -6 & -5 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -1$$

$$|A_0| = \begin{vmatrix} -6 & 3 & -3 & 0 \\ 55 & -18 & 10 & 8 \\ -34 & 11 & -6 & -5 \\ -1 & 0 & 1 & 0 \end{vmatrix} = 3$$

$$|A_1| = \begin{vmatrix} -1 & -6 & -3 & 0 \\ -3 & 55 & 10 & 8 \\ 2 & -34 & -6 & -5 \\ 0 & -1 & 1 & 0 \end{vmatrix} = 4$$

$$|A_2| = \begin{vmatrix} -1 & 3 & -6 & 0 \\ -3 & -18 & 55 & 8 \\ 2 & 11 & -34 & -5 \\ 0 & 0 & -1 & 0 \end{vmatrix} = 1$$

$$|A_3| = \begin{vmatrix} -1 & 3 & -3 & -6 \\ -3 & -18 & 10 & 55 \\ 2 & 11 & -6 & -34 \\ 0 & 0 & 1 & -1 \end{vmatrix} = 2$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{3}{-1} = -3 \\ x_1 = \frac{|A_1|}{|A|} = \frac{4}{-1} = -4 \\ x_2 = \frac{|A_2|}{|A|} = \frac{1}{-1} = -1 \\ x_3 = \frac{|A_3|}{|A|} = \frac{2}{-1} = -2 \end{cases}$$

A.4

$$|A| = \begin{vmatrix} 1 & -1 & -2 & 0 \\ -1 & 4 & 3 & -3 \\ -1 & 0 & -1 & 2 \\ 1 & -1 & 1 & -1 \end{vmatrix} = -1$$

$$|A_0| = \begin{vmatrix} -7 & -1 & -2 & 0 \\ 10 & 4 & 3 & -3 \\ -3 & 0 & -1 & 2 \\ 3 & -1 & 1 & -1 \end{vmatrix} = 2$$

$$|A_1| = \begin{vmatrix} 1 & -7 & -2 & 0 \\ -1 & 10 & 3 & -3 \\ -1 & -3 & -1 & 2 \\ 1 & 3 & 1 & -1 \end{vmatrix} = 1$$

$$|A_2| = \begin{vmatrix} 1 & -1 & -7 & 0 \\ -1 & 4 & 10 & -3 \\ -1 & 0 & -3 & 2 \\ 1 & -1 & 3 & -1 \end{vmatrix} = -3$$

$$|A_3| = \begin{vmatrix} 1 & -1 & -2 & -7 \\ -1 & 4 & 3 & 10 \\ -1 & 0 & -1 & -3 \\ 1 & -1 & 1 & 3 \end{vmatrix} = 1$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{2}{-1} = -2 \\ x_1 = \frac{|A_1|}{|A|} = \frac{1}{-1} = -1 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-3}{-1} = 3 \\ x_3 = \frac{|A_3|}{|A|} = \frac{1}{-1} = -1 \end{cases}$$

A.5

$$|A| = \begin{vmatrix} 1 & -3 & 1 \\ -1 & 4 & -2 \\ -3 & 4 & 3 \end{vmatrix} = 1$$

$$|A_0| = \begin{vmatrix} 4 & -3 & 1 \\ -8 & 4 & -2 \\ 10 & 4 & 3 \end{vmatrix} = -4$$

$$|A_1| = \begin{vmatrix} 1 & 4 & 1 \\ -1 & -8 & -2 \\ -3 & 10 & 3 \end{vmatrix} = -2$$

$$|A_2| = \begin{vmatrix} 1 & -3 & 4 \\ -1 & 4 & -8 \\ -3 & 4 & 10 \end{vmatrix} = 2$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{-4}{1} = -4 \\ x_1 = \frac{|A_1|}{|A|} = \frac{-2}{1} = -2 \\ x_2 = \frac{|A_2|}{|A|} = \frac{2}{1} = 2 \end{cases}$$

A.6

$$|A| = \begin{vmatrix} 8 & -11 & 0 \\ -13 & 18 & 0 \\ -7 & 9 & 1 \end{vmatrix} = 1$$

$$|A_0| = \begin{vmatrix} 36 & -11 & 0 \\ -59 & 18 & 0 \\ -32 & 9 & 1 \end{vmatrix} = -1$$

$$|A_1| = \begin{vmatrix} 8 & 36 & 0 \\ -13 & -59 & 0 \\ -7 & -32 & 1 \end{vmatrix} = -4$$

$$|A_2| = \begin{vmatrix} 8 & -11 & 36 \\ -13 & 18 & -59 \\ -7 & 9 & -32 \end{vmatrix} = -3$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{-1}{1} = -1 \\ x_1 = \frac{|A_1|}{|A|} = \frac{-4}{1} = -4 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-3}{1} = -3 \end{cases}$$

A.7

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ -3 & -21 & 13 \\ 4 & 34 & -21 \end{vmatrix} = -1$$

$$|A_0| = \begin{vmatrix} -4 & 0 & 0 \\ 7 & -21 & 13 \\ -8 & 34 & -21 \end{vmatrix} = 4$$

$$|A_1| = \begin{vmatrix} 1 & -4 & 0 \\ -3 & 7 & 13 \\ 4 & -8 & -21 \end{vmatrix} = 1$$

$$|A_2| = \begin{vmatrix} 1 & 0 & -4 \\ -3 & -21 & 7 \\ 4 & 34 & -8 \end{vmatrix} = 2$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{4}{-1} = -4 \\ x_1 = \frac{|A_1|}{|A|} = \frac{1}{-1} = -1 \\ x_2 = \frac{|A_2|}{|A|} = \frac{2}{-1} = -2 \end{cases}$$

A.8

$$|A| = \begin{vmatrix} 2 & -2 & 2 & 1 \\ -3 & 2 & -3 & 1 \\ 2 & -1 & 1 & -1 \\ -2 & 3 & 0 & -4 \end{vmatrix} = 1$$

$$|A_0| = \begin{vmatrix} 1 & -2 & 2 & 1 \\ 8 & 2 & -3 & 1 \\ -8 & -1 & 1 & -1 \\ -10 & 3 & 0 & -4 \end{vmatrix} = -4$$

$$|A_1| = \begin{vmatrix} 2 & 1 & 2 & 1 \\ -3 & 8 & -3 & 1 \\ 2 & -8 & 1 & -1 \\ -2 & -10 & 0 & -4 \end{vmatrix} = -2$$

$$|A_2| = \begin{vmatrix} 2 & -2 & 1 & 1 \\ -3 & 2 & 8 & 1 \\ 2 & -1 & -8 & -1 \\ -2 & 3 & -10 & -4 \end{vmatrix} = 1$$

$$|A_3| = \begin{vmatrix} 2 & -2 & 2 & 1 \\ -3 & 2 & -3 & 8 \\ 2 & -1 & 1 & -8 \\ -2 & 3 & 0 & -10 \end{vmatrix} = 3$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{-4}{1} = -4 \\ x_1 = \frac{|A_1|}{|A|} = \frac{-2}{1} = -2 \\ x_2 = \frac{|A_2|}{|A|} = \frac{1}{1} = 1 \\ x_3 = \frac{|A_3|}{|A|} = \frac{3}{1} = 3 \end{cases}$$

A.9

$$|A| = \begin{vmatrix} -5 & -3 & 2 & 5 \\ -2 & -1 & 1 & 0 \\ 4 & 3 & -2 & -2 \\ 2 & -1 & 0 & -3 \end{vmatrix} = 1$$

$$|A_0| = \begin{vmatrix} 13 & -3 & 2 & 5 \\ 0 & -1 & 1 & 0 \\ -3 & 3 & -2 & -2 \\ -14 & -1 & 0 & -3 \end{vmatrix} = -1$$

$$|A_1| = \begin{vmatrix} -5 & 13 & 2 & 5 \\ -2 & 0 & 1 & 0 \\ 4 & -3 & -2 & -2 \\ 2 & -14 & 0 & -3 \end{vmatrix} = 3$$

$$|A_2| = \begin{vmatrix} -5 & -3 & 13 & 5 \\ -2 & -1 & 0 & 0 \\ 4 & 3 & -3 & -2 \\ 2 & -1 & -14 & -3 \end{vmatrix} = 1$$

$$|A_3| = \begin{vmatrix} -5 & -3 & 2 & 13 \\ -2 & -1 & 1 & 0 \\ 4 & 3 & -2 & -3 \\ 2 & -1 & 0 & -14 \end{vmatrix} = 3$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{-1}{1} = -1 \\ x_1 = \frac{|A_1|}{|A|} = \frac{3}{1} = 3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{1}{1} = 1 \\ x_3 = \frac{|A_3|}{|A|} = \frac{3}{1} = 3 \end{cases}$$

A.10

$$|A| = \begin{vmatrix} -3 & 5 & -5 \\ 9 & -10 & 8 \\ -2 & 1 & 0 \end{vmatrix} = -1$$

$$|A_0| = \begin{vmatrix} -21 & 5 & -5 \\ 27 & -10 & 8 \\ 3 & 1 & 0 \end{vmatrix} = 3$$

$$|A_1| = \begin{vmatrix} -3 & -21 & -5 \\ 9 & 27 & 8 \\ -2 & 3 & 0 \end{vmatrix} = 3$$

$$|A_2| = \begin{vmatrix} -3 & 5 & -21 \\ 9 & -10 & 27 \\ -2 & 1 & 3 \end{vmatrix} = -3$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{3}{-1} = -3 \\ x_1 = \frac{|A_1|}{|A|} = \frac{3}{-1} = -3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-3}{-1} = 3 \end{cases}$$

A.11

$$|A| = \begin{vmatrix} 0 & 2 & -1 \\ -3 & 4 & -1 \\ 5 & -7 & 2 \end{vmatrix} = 1$$

$$|A_0| = \begin{vmatrix} -7 & 2 & -1 \\ -22 & 4 & -1 \\ 38 & -7 & 2 \end{vmatrix} = 3$$

$$|A_1| = \begin{vmatrix} 0 & -7 & -1 \\ -3 & -22 & -1 \\ 5 & 38 & 2 \end{vmatrix} = -3$$

$$|A_2| = \begin{vmatrix} 0 & 2 & -7 \\ -3 & 4 & -22 \\ 5 & -7 & 38 \end{vmatrix} = 1$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{3}{1} = 3 \\ x_1 = \frac{|A_1|}{|A|} = \frac{-3}{1} = -3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{1}{1} = 1 \end{cases}$$

A.12

$$|A| = \begin{vmatrix} -1 & -1 & 1 \\ 3 & -2 & -1 \\ -5 & 8 & 0 \end{vmatrix} = 1$$

$$|A_0| = \begin{vmatrix} 5 & -1 & 1 \\ -6 & -2 & -1 \\ 2 & 8 & 0 \end{vmatrix} = -2$$

$$|A_1| = \begin{vmatrix} -1 & 5 & 1 \\ 3 & -6 & -1 \\ -5 & 2 & 0 \end{vmatrix} = -1$$

$$|A_2| = \begin{vmatrix} -1 & -1 & 5 \\ 3 & -2 & -6 \\ -5 & 8 & 2 \end{vmatrix} = 2$$

$$\begin{cases} x_0 = \frac{|A_0|}{|A|} = \frac{-2}{1} = -2 \\ x_1 = \frac{|A_1|}{|A|} = \frac{-1}{1} = -1 \\ x_2 = \frac{|A_2|}{|A|} = \frac{2}{1} = 2 \end{cases}$$