代幾 I 計算演習 [問題] (2008/10/02)

問. 次の二つの複素ベクトル u, v の内積 (u, v) を求めなさい

Q.1

$$\mathrm{u} = \begin{pmatrix} 1+i \\ -2+3i \\ 2+i \\ 2+i \\ 2 \\ -3i \\ 3-2i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} 1+i \\ 2i \\ -2-2i \\ -3-3i \\ -1+2i \\ 0 \\ 2i \end{pmatrix}$$

Q.2

$$\mathrm{u} = \begin{pmatrix} 2-2i \\ 2i \\ 3-i \\ -2+i \\ -2i \\ 3+3i \\ 0 \end{pmatrix}, \mathrm{v} = \begin{pmatrix} 1 \\ -3-2i \\ -1-3i \\ -1+3i \\ i \\ 3+2i \\ -2-2i \end{pmatrix}$$

Q.3

$$\mathrm{u} = \begin{pmatrix} 1+3i \\ -1-2i \\ -1 \\ 3-3i \\ 3i \\ 3+i \\ -3i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} -1+3i \\ -2-i \\ 1+i \\ 3-i \\ -3+3i \\ -3+i \\ 2+i \end{pmatrix}$$

Q.4

$$\mathrm{u} = \begin{pmatrix} -1-i \\ -2+i \\ 3+3i \\ 1+3i \\ -3 \\ -3+i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} 1-3i \\ 2 \\ -2+3i \\ -1 \\ -3-2i \\ 1+i \end{pmatrix}$$

Q.5

$$\mathrm{u} = \begin{pmatrix} -1 \\ 3-3i \\ 2-i \\ -1 \\ -2-2i \\ -3-i \\ -3-i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} -2+2i \\ 1-3i \\ -1+2i \\ -1 \\ -i \\ 3-2i \\ 2-3i \end{pmatrix}$$

Q.6

$$\mathrm{u} = \begin{pmatrix} -3-3i \\ 2 \\ 3+i \\ -2-2i \\ -3i \\ 0 \\ -3+3i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} -2-3i \\ i \\ -1+3i \\ -2+2i \\ 2-3i \\ -1-2i \\ -3-i \end{pmatrix}$$

代幾 I 計算演習 [解答] (2008/10/02)

A.1

$$
\begin{aligned}
\left(\begin{pmatrix} 1+i \\ -2+3i \\ 2+i \\ 2+i \\ 2 \\ -3i \\ 3-2i \end{pmatrix}, \begin{pmatrix} 1+i \\ 2i \\ -2-2i \\ -3-3i \\ -1+2i \\ 0 \\ 2i \end{pmatrix}\right) &= (1+i)\times\overline{(1+i)} + (-2+3i)\times\overline{(2i)} \\
&+ (2+i)\times\overline{(-2-2i)} + (2+i)\times\overline{(-3-3i)} \\
&+ (2)\times\overline{(-1+2i)} + (-3i)\times\overline{(0)} \\
&+ (3-2i)\times\overline{(2i)} \\
&= (1+i)\times(1-i) + (-2+3i)\times(-2i) \\
&+ (2+i)\times(-2+2i) + (2+i)\times(-3+3i) \\
&+ (2)\times(-1-2i) + (-3i)\times(0) \\
&+ (3-2i)\times(-2i) \\
&= (2) + (6+4i) + (-6+2i) + (-9+3i) \\
&+ (-2-4i) + (0) + (-4-6i) \\
&= -13-i
\end{aligned}
$$

A.2

$$
\begin{aligned}
\left(\begin{pmatrix} 2-2i \\ 2i \\ 3-i \\ -2+i \\ -2i \\ 3+3i \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -3-2i \\ -1-3i \\ -1+3i \\ i \\ 3+2i \\ -2-2i \end{pmatrix}\right) &= (2-2i)\times\overline{(1)} + (2i)\times\overline{(-3-2i)} \\
&+ (3-i)\times\overline{(-1-3i)} + (-2+i)\times\overline{(-1+3i)} \\
&+ (-2i)\times\overline{(i)} + (3+3i)\times\overline{(3+2i)} \\
&+ (0)\times\overline{(-2-2i)} \\
&= (2-2i)\times(1) + (2i)\times(-3+2i) \\
&+ (3-i)\times(-1+3i) + (-2+i)\times(-1-3i) \\
&+ (-2i)\times(-i) + (3+3i)\times(3-2i) \\
&+ (0)\times(-2+2i) \\
&= (2-2i) + (-4-6i) + (10i) + (5+5i) \\
&+ (-2) + (15+3i) + (0) \\
&= 16+10i
\end{aligned}
$$

A.3

$$
\begin{aligned}
\left(\begin{pmatrix} 1+3i \\ -1-2i \\ -1 \\ 3-3i \\ 3i \\ 3+i \\ -3i \end{pmatrix}, \begin{pmatrix} -1+3i \\ -2-i \\ 1+i \\ 3-i \\ -3+3i \\ -3+i \\ 2+i \end{pmatrix}\right) &= (1+3i) \times \overline{(-1+3i)} + (-1-2i) \times \overline{(-2-i)} \\
&+ (-1) \times \overline{(1+i)} + (3-3i) \times \overline{(3-i)} \\
&+ (3i) \times \overline{(-3+3i)} + (3+i) \times \overline{(-3+i)} \\
&+ (-3i) \times \overline{(2+i)} \\
&= (1+3i) \times (-1-3i) + (-1-2i) \times (-2+i) \\
&+ (-1) \times (1-i) + (3-3i) \times (3+i) \\
&+ (3i) \times (-3-3i) + (3+i) \times (-3-i) \\
&+ (-3i) \times (2-i) \\
&= (8-6i) + (4+3i) + (-1+i) + (12-6i) \\
&+ (9-9i) + (-8-6i) + (-3-6i) \\
&= 21-29i
\end{aligned}
$$

A.4

$$
\begin{aligned}
\left(\begin{pmatrix} -1-i \\ -2+i \\ 3+3i \\ 1+3i \\ -3 \\ -3+i \end{pmatrix}, \begin{pmatrix} 1-3i \\ 2 \\ -2+3i \\ -1 \\ -3-2i \\ 1+i \end{pmatrix}\right) &= (-1-i) \times \overline{(1-3i)} + (-2+i) \times \overline{(2)} \\
&+ (3+3i) \times \overline{(-2+3i)} + (1+3i) \times \overline{(-1)} \\
&+ (-3) \times \overline{(-3-2i)} + (-3+i) \times \overline{(1+i)} \\
&= (-1-i) \times (1+3i) + (-2+i) \times (2) \\
&+ (3+3i) \times (-2-3i) + (1+3i) \times (-1) \\
&+ (-3) \times (-3+2i) + (-3+i) \times (1-i) \\
&= (2-4i) + (-4+2i) + (3-15i) + (-1-3i) \\
&+ (9-6i) + (-2+4i) \\
&= 7-22i
\end{aligned}
$$

A.5

$$
\begin{aligned}
\left(\begin{pmatrix} -1 \\ 3-3i \\ 2-i \\ -1 \\ -2-2i \\ -3-i \\ -3-i \end{pmatrix}, \begin{pmatrix} -2+2i \\ 1-3i \\ -1+2i \\ -1 \\ -i \\ 3-2i \\ 2-3i \end{pmatrix}\right) &= (-1) \times \overline{(-2+2i)} + (3-3i) \times \overline{(1-3i)} \\
&+ (2-i) \times \overline{(-1+2i)} + (-1) \times \overline{(-1)} \\
&+ (-2-2i) \times \overline{(-i)} + (-3-i) \times \overline{(3-2i)} \\
&+ (-3-i) \times \overline{(2-3i)} \\
&= (-1) \times (-2-2i) + (3-3i) \times (1+3i) \\
&+ (2-i) \times (-1-2i) + (-1) \times (-1) \\
&+ (-2-2i) \times (i) + (-3-i) \times (3+2i) \\
&+ (-3-i) \times (2+3i) \\
&= (2+2i) + (12+6i) + (-4-3i) + (1) \\
&+ (2-2i) + (-7-9i) + (-3-11i) \\
&= 3-17i
\end{aligned}
$$

A.6

$$
\begin{aligned}
\left(\begin{pmatrix} -3-3i \\ 2 \\ 3+i \\ -2-2i \\ -3i \\ 0 \\ -3+3i \end{pmatrix}, \begin{pmatrix} -2-3i \\ i \\ -1+3i \\ -2+2i \\ 2-3i \\ -1-2i \\ -3-i \end{pmatrix}\right) &= (-3-3i) \times \overline{(-2-3i)} + (2) \times \overline{(i)} \\
&+ (3+i) \times \overline{(-1+3i)} + (-2-2i) \times \overline{(-2+2i)} \\
&+ (-3i) \times \overline{(2-3i)} + (0) \times \overline{(-1-2i)} \\
&+ (-3+3i) \times \overline{(-3-i)} \\
&= (-3-3i) \times (-2+3i) + (2) \times (-i) \\
&+ (3+i) \times (-1-3i) + (-2-2i) \times (-2-2i) \\
&+ (-3i) \times (2+3i) + (0) \times (-1+2i) \\
&+ (-3+3i) \times (-3+i) \\
&= (15-3i) + (-2i) + (-10i) + (8i) \\
&+ (9-6i) + (0) + (6-12i) \\
&= 30-25i
\end{aligned}
$$