# 代幾 I 計算演習 [問題] (2008/11/06)

問. 次の行列の行列式を求めなさい

Q.1

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 3 & 9 \end{vmatrix}$$

Q.2

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & -5 & 5 & -4 \\ 1 & 25 & 25 & 16 \\ -1 & -125 & 125 & -64 \end{vmatrix}$$

Q.3

$$\begin{vmatrix} 1 & -2 & 4 & -8 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & -5 & 25 & -125 \end{vmatrix}$$

Q.4

$$\begin{vmatrix} 1 & 2 & 4 & 8 \\ 1 & 5 & 25 & 125 \\ 1 & -5 & 25 & -125 \\ 1 & -1 & 1 & -1 \end{vmatrix}$$

Q.5

$$\begin{vmatrix} 1 & -2 & 4 \\ 1 & 2 & 4 \\ 1 & 4 & 16 \end{vmatrix}$$

Q.6

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & -1 \\ 16 & 4 & 1 \end{vmatrix}$$

Q.7

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \\ 1 & -5 & 25 & -125 \end{vmatrix}$$

Q.8

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 5 & -3 & -2 & 4 \\ 25 & 9 & 4 & 16 \\ 125 & -27 & -8 & 64 \end{vmatrix}$$

Q.9

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & -3 & -1 \\ 9 & 9 & 1 \end{vmatrix}$$

Q.10

$$\begin{vmatrix} 1 & -5 & 25 \\ 1 & -3 & 9 \\ 1 & -4 & 16 \end{vmatrix}$$

Q.11

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & -2 & 4 & -3 \\ 4 & 4 & 16 & 9 \\ 8 & -8 & 64 & -27 \end{vmatrix}$$

Q.12

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 4 \\ 1 & 4 & 1 & 16 \\ 1 & 8 & -1 & 64 \end{vmatrix}$$

# 代幾 I 計算演習 [解答] (2008/11/06)

A.1

$$
\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 3 & 9 \end{vmatrix} \\
&= \begin{vmatrix} 1^0 & 1^1 & 1^2 \\ 0^0 & 0^1 & 0^2 \\ 3^0 & 3^1 & 3^2 \end{vmatrix} \\
&= (3-0)\times(3-1) \\
&\times\ (0-1) \\
&= -6
\end{aligned}
$$

A.3

$$
\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & -2 & 4 & -8 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & -5 & 25 & -125 \end{vmatrix} \\
&= \begin{vmatrix} (-2)^0 & (-2)^1 & (-2)^2 & (-2)^3 \\ 1^0 & 1^1 & 1^2 & 1^3 \\ 0^0 & 0^1 & 0^2 & 0^3 \\ (-5)^0 & (-5)^1 & (-5)^2 & (-5)^3 \end{vmatrix} \\
&= (-5-0)\times(-5-1)\times(-5-(-2)) \\
&\times\ (0-1)\times(0-(-2)) \\
&\times\ (1-(-2)) \\
&= 540
\end{aligned}
$$

A.2

$$
\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & -5 & 5 & -4 \\ 1 & 25 & 25 & 16 \\ -1 & -125 & 125 & -64 \end{vmatrix} \\
&= \begin{vmatrix} (-1)^0 & (-5)^0 & 5^0 & (-4)^0 \\ (-1)^1 & (-5)^1 & 5^1 & (-4)^1 \\ (-1)^2 & (-5)^2 & 5^2 & (-4)^2 \\ (-1)^3 & (-5)^3 & 5^3 & (-4)^3 \end{vmatrix} \\
&= (-4-5)\times(-4-(-5))\times(-4-(-1)) \\
&\times\ (5-(-5))\times(5-(-1)) \\
&\times\ (-5-(-1)) \\
&= -6480
\end{aligned}
$$

A.4

$$
\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & 2 & 4 & 8 \\ 1 & 5 & 25 & 125 \\ 1 & -5 & 25 & -125 \\ 1 & -1 & 1 & -1 \end{vmatrix} \\
&= \begin{vmatrix} 2^0 & 2^1 & 2^2 & 2^3 \\ 5^0 & 5^1 & 5^2 & 5^3 \\ (-5)^0 & (-5)^1 & (-5)^2 & (-5)^3 \\ (-1)^0 & (-1)^1 & (-1)^2 & (-1)^3 \end{vmatrix} \\
&= (-1-(-5))\times(-1-5)\times(-1-2) \\
&\times\ (-5-5)\times(-5-2) \\
&\times\ (5-2) \\
&= 15120
\end{aligned}
$$

A.5

$$
\begin{aligned}
与式 &= \begin{vmatrix} 1 & -2 & 4 \\ 1 & 2 & 4 \\ 1 & 4 & 16 \end{vmatrix} \\
&= \begin{vmatrix} (-2)^0 & (-2)^1 & (-2)^2 \\ 2^0 & 2^1 & 2^2 \\ 4^0 & 4^1 & 4^2 \end{vmatrix} \\
&= (4-2)\times(4-(-2)) \\
&\quad \times (2-(-2)) \\
&= 48
\end{aligned}
$$

A.6

$$
\begin{aligned}
与式 &= \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & -1 \\ 16 & 4 & 1 \end{vmatrix} \\
&= \begin{vmatrix} 4^0 & 2^0 & (-1)^0 \\ 4^1 & 2^1 & (-1)^1 \\ 4^2 & 2^2 & (-1)^2 \end{vmatrix} \\
&= (-1-2)\times(-1-4) \\
&\quad \times (2-4) \\
&= -30
\end{aligned}
$$

A.7

$$
\begin{aligned}
与式 &= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \\ 1 & -5 & 25 & -125 \end{vmatrix} \\
&= \begin{vmatrix} 0^0 & 0^1 & 0^2 & 0^3 \\ 4^0 & 4^1 & 4^2 & 4^3 \\ 5^0 & 5^1 & 5^2 & 5^3 \\ (-5)^0 & (-5)^1 & (-5)^2 & (-5)^3 \end{vmatrix} \\
&= (-5-5)\times(-5-4)\times(-5-0) \\
&\quad \times (5-4)\times(5-0) \\
&\quad \times (4-0) \\
&= -9000
\end{aligned}
$$

A.8

$$
\begin{aligned}
与式 &= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 5 & -3 & -2 & 4 \\ 25 & 9 & 4 & 16 \\ 125 & -27 & -8 & 64 \end{vmatrix} \\
&= \begin{vmatrix} 5^0 & (-3)^0 & (-2)^0 & 4^0 \\ 5^1 & (-3)^1 & (-2)^1 & 4^1 \\ 5^2 & (-3)^2 & (-2)^2 & 4^2 \\ 5^3 & (-3)^3 & (-2)^3 & 4^3 \end{vmatrix} \\
&= (4-(-2))\times(4-(-3))\times(4-5) \\
&\quad \times (-2-(-3))\times(-2-5) \\
&\quad \times (-3-5) \\
&= -2352
\end{aligned}
$$

A.9

$$
\begin{aligned}
与式 &= \begin{vmatrix} 1 & 1 & 1 \\ 3 & -3 & -1 \\ 9 & 9 & 1 \end{vmatrix} \\
&= \begin{vmatrix} 3^0 & (-3)^0 & (-1)^0 \\ 3^1 & (-3)^1 & (-1)^1 \\ 3^2 & (-3)^2 & (-1)^2 \end{vmatrix} \\
&= (-1-(-3))\times(-1-3) \\
&\quad \times (-3-3) \\
&= 48
\end{aligned}
$$

A.10

$$
\begin{aligned}
与式 &= \begin{vmatrix} 1 & -5 & 25 \\ 1 & -3 & 9 \\ 1 & -4 & 16 \end{vmatrix} \\
&= \begin{vmatrix} (-5)^0 & (-5)^1 & (-5)^2 \\ (-3)^0 & (-3)^1 & (-3)^2 \\ (-4)^0 & (-4)^1 & (-4)^2 \end{vmatrix} \\
&= (-4-(-3))\times(-4-(-5)) \\
&\quad \times (-3-(-5)) \\
&= -2
\end{aligned}
$$

A.11

$$
\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & -2 & 4 & -3 \\ 4 & 4 & 16 & 9 \\ 8 & -8 & 64 & -27 \end{vmatrix} \\
&= \begin{vmatrix} 2^0 & (-2)^0 & 4^0 & (-3)^0 \\ 2^1 & (-2)^1 & 4^1 & (-3)^1 \\ 2^2 & (-2)^2 & 4^2 & (-3)^2 \\ 2^3 & (-2)^3 & 4^3 & (-3)^3 \end{vmatrix} \\
&= (-3-4) \times (-3-(-2)) \times (-3-2) \\
&\quad \times (4-(-2)) \times (4-2) \\
&\quad \times (-2-2) \\
&= 1680
\end{aligned}
$$

A.12

$$
\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 4 \\ 1 & 4 & 1 & 16 \\ 1 & 8 & -1 & 64 \end{vmatrix} \\
&= \begin{vmatrix} 1^0 & 2^0 & (-1)^0 & 4^0 \\ 1^1 & 2^1 & (-1)^1 & 4^1 \\ 1^2 & 2^2 & (-1)^2 & 4^2 \\ 1^3 & 2^3 & (-1)^3 & 4^3 \end{vmatrix} \\
&= (4-(-1)) \times (4-2) \times (4-1) \\
&\quad \times (-1-2) \times (-1-1) \\
&\quad \times (2-1) \\
&= 180
\end{aligned}
$$