代幾 I 計算演習 [問題] (2008/12/18)

問. 次の独立なベクトルから、シュミットの直交化を利用して、正規直交系を求めなさい

Q.1

$$< \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} >$$

Q.2

$$< \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} >$$

Q.3

$$< \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} >$$

Q.4

$$< \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} >$$

Q.5

$$< \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} >$$

Q.6

$$< \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ 3 \end{pmatrix} >$$

Q.7

$$< \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} >$$

Q.8

$$< \begin{pmatrix} 0 \\ 2 \\ -5 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} >$$

Q.9

$$< \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} >$$

Q.10

$$< \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} >$$

Q.11

$$< \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} >$$

Q.12

$$< \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -5 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} >$$

Q.13

$$< \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} >$$

Q.14

$$< \begin{pmatrix} -5 \\ 7 \\ -5 \end{pmatrix}, \begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} >$$

代幾 I 計算演習 [解答] (2008/12/18)

A.1

$$< \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{pmatrix} >$$

A.2

$$< \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6} \end{pmatrix} >$$

A.3

$$< \begin{pmatrix} 0 \\ -\frac{2\sqrt{13}}{13} \\ \frac{3\sqrt{13}}{13} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{1742}}{134} \\ \frac{33\sqrt{1742}}{1742} \\ \frac{11\sqrt{1742}}{871} \end{pmatrix}, \begin{pmatrix} \frac{11\sqrt{134}}{134} \\ -\frac{3\sqrt{134}}{134} \\ -\frac{\sqrt{134}}{67} \end{pmatrix} >$$

A.4

$$< \begin{pmatrix} \frac{2\sqrt{5}}{5} \\ 0 \\ -\frac{\sqrt{5}}{5} \end{pmatrix}, \begin{pmatrix} \frac{2\sqrt{5}}{15} \\ \frac{\sqrt{5}}{3} \\ \frac{4\sqrt{5}}{15} \end{pmatrix}, \begin{pmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{pmatrix} >$$

A.5

$$< \begin{pmatrix} \frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{30}}{30} \\ \frac{\sqrt{30}}{15} \\ \frac{\sqrt{30}}{6} \end{pmatrix}, \begin{pmatrix} -\frac{2\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \\ 0 \end{pmatrix} >$$

A.6

$$< \begin{pmatrix} 0 \\ \frac{2\sqrt{5}}{5} \\ -\frac{\sqrt{5}}{5} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{5}}{3} \\ \frac{2\sqrt{5}}{15} \\ \frac{4\sqrt{5}}{15} \end{pmatrix}, \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} >$$

A.7

$$< \begin{pmatrix} -\frac{\sqrt{14}}{14} \\ -\frac{3\sqrt{14}}{14} \\ \frac{\sqrt{14}}{7} \end{pmatrix}, \begin{pmatrix} \frac{11\sqrt{266}}{266} \\ -\frac{9\sqrt{266}}{266} \\ -\frac{4\sqrt{266}}{133} \end{pmatrix}, \begin{pmatrix} \frac{3\sqrt{19}}{19} \\ \frac{\sqrt{19}}{19} \\ \frac{3\sqrt{19}}{19} \end{pmatrix} >$$

A.8

$$< \begin{pmatrix} 0 \\ \frac{2\sqrt{29}}{29} \\ -\frac{5\sqrt{29}}{29} \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{5\sqrt{29}}{29} \\ \frac{2\sqrt{29}}{29} \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} >$$

A.9

$$< \begin{pmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{6} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{30}}{15} \\ -\frac{\sqrt{30}}{6} \\ -\frac{\sqrt{30}}{30} \end{pmatrix} >$$

A.10

$$< \begin{pmatrix} \frac{\sqrt{5}}{5} \\ -\frac{2\sqrt{5}}{5} \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{2\sqrt{5}}{5} \\ -\frac{\sqrt{5}}{5} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} >$$

A.11

$$< \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{6} \\ \frac{2\sqrt{2}}{3} \\ \frac{\sqrt{2}}{6} \end{pmatrix}, \begin{pmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{pmatrix} >$$

A.12

$$< \begin{pmatrix} \frac{3\sqrt{14}}{14} \\ -\frac{\sqrt{14}}{14} \\ \frac{\sqrt{14}}{7} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{70}}{14} \\ \frac{3\sqrt{70}}{70} \\ -\frac{3\sqrt{70}}{35} \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{2\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \end{pmatrix} >$$

A.13

$$< \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix} >$$

A.14

$$< \begin{pmatrix} -\frac{5\sqrt{11}}{33} \\ \frac{7\sqrt{11}}{33} \\ -\frac{5\sqrt{11}}{33} \end{pmatrix}, \begin{pmatrix} \frac{7\sqrt{22}}{66} \\ \frac{5\sqrt{22}}{33} \\ \frac{7\sqrt{22}}{66} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{pmatrix} >$$