代幾 I 計算演習 [問題] (2009/01/15)

問. 次の独立な複素ベクトルから、シュミットの直交化を利用して、正規直交系を求めなさい

Q.1

$$< \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \begin{pmatrix} -i \\ 1+i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} >$$

Q.2

$$< \begin{pmatrix} 0 \\ 1 \\ -i \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ i \end{pmatrix} >$$

Q.3

$$< \begin{pmatrix} -i \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1-i \\ 0 \\ 1 \end{pmatrix} >$$

Q.4

$$< \begin{pmatrix} -2i \\ i \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ -i \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix} >$$

Q.5

$$< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -i \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} >$$

Q.6

$$< \begin{pmatrix} -1 \\ -2i \\ 2-i \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -i \\ 1 \\ 1+i \end{pmatrix} >$$

Q.7

$$< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1+i \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2i \\ -i \\ 0 \end{pmatrix} >$$

Q.8

$$< \begin{pmatrix} 1 \\ 0 \\ -i \end{pmatrix}, \begin{pmatrix} i \\ -i \\ 1+i \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ i \end{pmatrix} >$$

Q.9

$$< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} i \\ 1 \\ i \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ i \end{pmatrix} >$$

Q.10

$$< \begin{pmatrix} -1 \\ -i \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1+i \\ 1-i \\ 1 \end{pmatrix} >$$

Q.11

$$< \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ i \\ -1 \end{pmatrix}, \begin{pmatrix} i \\ -1 \\ -i \end{pmatrix} >$$

Q.12

$$< \begin{pmatrix} i \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ i \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} >$$

Q.13

$$< \begin{pmatrix} -1 \\ -2i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix} >$$

Q.14

$$< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2i \\ -i \\ i \end{pmatrix} >$$

代幾 I 計算演習 [解答] (2009/01/15)

A.1

$$< \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2}i \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2}i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} >$$

A.2

$$< \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2}i \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6}i \\ -\frac{\sqrt{6}}{6} \end{pmatrix}, \begin{pmatrix} i \\ 1 \\ i \end{pmatrix} >$$

A.3

$$< \begin{pmatrix} -i \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{2\sqrt{3}}{3}i \\ \frac{\sqrt{3}}{3} \\ 0 \end{pmatrix}, \begin{pmatrix} -10-10i \\ -2+2i \\ 3 \end{pmatrix} >$$

A.4

$$< \begin{pmatrix} -\sqrt{2}i \\ \frac{\sqrt{2}}{2}i \\ \frac{\sqrt{2}}{2} \end{pmatrix}, \begin{pmatrix} -\frac{5\sqrt{134}}{67} \\ \frac{3\sqrt{134}}{134} \\ -\frac{5\sqrt{134}}{134}i \end{pmatrix}, \begin{pmatrix} \frac{59\sqrt{7571}}{7571} \\ -\frac{11\sqrt{7571}}{7571} \\ \frac{63\sqrt{7571}}{7571}i \end{pmatrix} >$$

A.5

$$< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} >$$

A.6

$$< \begin{pmatrix} -\frac{\sqrt{10}}{10} \\ -\frac{\sqrt{10}}{5}i \\ \frac{\sqrt{10}}{5} - \frac{\sqrt{10}}{10}i \end{pmatrix}, \begin{pmatrix} \frac{3\sqrt{10}}{10} \\ -\frac{\sqrt{10}}{15}i \\ \frac{\sqrt{10}}{15} - \frac{\sqrt{10}}{30}i \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{1}{3} + \frac{2}{3}i \\ \frac{2}{3} \end{pmatrix} >$$

A.7

$$< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{\sqrt{2}}{2}i \\ \frac{\sqrt{2}}{2}i \end{pmatrix} >$$

A.8

$$< \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2}i \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{3}i \\ \frac{\sqrt{6}}{6}i \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3}i \\ \frac{\sqrt{3}}{3}i \end{pmatrix} >$$

A.9

$$< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2}i \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2}i \end{pmatrix} >$$

A.10

$$< \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2}i \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2}i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} >$$

A.11

$$< \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ i \\ 0 \end{pmatrix}, \begin{pmatrix} i \\ 0 \\ 0 \end{pmatrix} >$$

A.12

$$< \begin{pmatrix} i \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ i \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{2\sqrt{5}}{5} \\ 0 \\ \frac{\sqrt{5}}{5} \end{pmatrix} >$$

A.13

$$< \begin{pmatrix} -\frac{\sqrt{5}}{5} \\ -\frac{2\sqrt{5}}{5}i \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{22}}{11}i \\ -\frac{\sqrt{22}}{22} \\ \frac{5\sqrt{22}}{22} \end{pmatrix}, \begin{pmatrix} -\frac{37\sqrt{1518}}{759}i \\ -\frac{37\sqrt{1518}}{1518} \\ \frac{25\sqrt{1518}}{506} \end{pmatrix} >$$

A.14

$$< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ i \end{pmatrix} >$$