

代数学幾何学 A/B 計算演習 (2009/04/16)

問 1. $\alpha = 1 + i, \beta = 2 - 3i$ として、次の計算をなさい。

1. $\alpha + \beta$

2. $\alpha - \beta$

3. $\alpha \times \beta$

4. $\frac{\alpha}{\beta}$

5. $\operatorname{Re}(\beta)$

6. $\operatorname{Im}(\beta)$

7. $\bar{\beta}$

8. $|\beta|$

問 2. 次の α, β に対して、問 1 と同様の計算を行いなさい。

1. $\alpha = -5 + 4i, \beta = -5 - i$

2. $\alpha = -2 - 5i, \beta = -5 - 5i$

3. $\alpha = -4 + i, \beta = 2 - 3i$

4. $\alpha = -5 - i, \beta = -3 + 5i$

5. $\alpha = 2 - 3i, \beta = 5 - i$

6. $\alpha = 2 + 5i, \beta = -2 + i$

7. $\alpha = 5 + 4i, \beta = 3 + i$

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問 1. $\alpha = 1 + i, \beta = 2 - 3i$ として、次の計算をなさい。

1. Q. $\alpha + \beta$

A.

$$\begin{aligned}\alpha + \beta &= (1 + i) + (2 - 3i) \\ &= (1 + 1) + (2 + (-3)i) \\ &= (1 + 2) + (1 + (-3))i \\ &= 3 + (-2)i \\ &= 3 - 2i\end{aligned}$$

2. Q. $\alpha - \beta$

A.

$$\begin{aligned}\alpha - \beta &= (1 + i) - (2 - 3i) \\ &= (1 + 1i) - (2 + (-3)i) \\ &= (1 - 2) + (1 - (-3))i \\ &= (-1) + (4)i \\ &= -1 + 4i\end{aligned}$$

3. Q. $\alpha \times \beta$

A.

$$\begin{aligned}\alpha \times \beta &= (1 + i) \times (2 - 3i) \\ &= 1 \times (2 - 3i) + i \times (2 - 3i) \\ &= (1 \times 2) - (1 \times 3i) + (i \times 2) - (i \times 3i) \\ &= 2 - 3i + 2i - 3i^2 \\ &= 2 - 3i + 2i - 3(-1) \\ &= 2 - 3i + 2i + 3 \\ &= (2 + 3) + (-3i + 2i) \\ &= (2 + 3) + (-3 + 2)i \\ &= 5 + (-1)i \\ &= 5 - i\end{aligned}$$

4. Q. $\frac{\alpha}{\beta}$

A.

$$\begin{aligned}\frac{\alpha}{\beta} &= \frac{(1+i)}{(2-3i)} \\ &= \frac{(1+i)\overline{(2-3i)}}{(2-3i)\overline{(2-3i)}} \\ &= \frac{(1+i)(2+3i)}{(2-3i)(2+3i)} \\ &= \frac{2+3i+2i-3}{2^2+(-3)^2} \\ &= \frac{-1+5i}{13} \\ &= -\frac{1}{13} + \frac{5}{13}i\end{aligned}$$

5. **Q.** $\operatorname{Re}(\beta)$

A.

$$\begin{aligned}\operatorname{Re}(\beta) &= \operatorname{Re}(2-3i) \\ &= 2\end{aligned}$$

6. **Q.** $\operatorname{Im}(\beta)$

A.

$$\begin{aligned}\operatorname{Im}(\beta) &= \operatorname{Im}(2-3i) \\ &= -3\end{aligned}$$

7. **Q.** $\bar{\beta}$

A.

$$\begin{aligned}\bar{\beta} &= \overline{2-3i} \\ &= 2+3i\end{aligned}$$

8. **Q.** $|\beta|$

A.

$$\begin{aligned}|\beta| &= \sqrt{2^2+3^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13}\end{aligned}$$

問 2. 次の α, β に対して、問 1 と同様の計算を行いなさい。

1. **Q.** $\alpha = -5 + 4i, \quad \beta = -5 - i$

A. 1-1. $-10 + 3i, \quad 1-2. \quad 5i, \quad 1-3. \quad 29 - 15i, \quad 1-4. \quad \frac{21-25i}{26}, \quad 1-5. \quad -5, \quad 1-6. \quad -1, \quad 1-7. \quad -5 + i, \quad 1-8. \quad \sqrt{26}$

2. **Q.** $\alpha = -2 - 5i, \quad \beta = -5 - 5i$

A. 2-1. $-7 - 10i, \quad 2-2. \quad 3, \quad 2-3. \quad -15 + 35i, \quad 2-4. \quad \frac{7+3i}{10}, \quad 2-5. \quad -5, \quad 2-6. \quad -5, \quad 2-7. \quad -5 + 5i, \quad 2-8. \quad 5\sqrt{2}$

3. **Q.** $\alpha = -4 + i, \quad \beta = 2 - 3i$

A. 3-1. $-2 - 2i, \quad 3-2. \quad -6 + 4i, \quad 3-3. \quad -5 + 14i, \quad 3-4. \quad \frac{-11-10i}{13}, \quad 3-5. \quad 2, \quad 3-6. \quad -3, \quad 3-7. \quad 2 + 3i, \quad 3-8. \quad \sqrt{13}$

4. **Q.** $\alpha = -5 - i, \quad \beta = -3 + 5i$

A. 4-1. $-8 + 4i, \quad 4-2. \quad -2 - 6i, \quad 4-3. \quad 20 - 22i, \quad 4-4. \quad \frac{5+14i}{17}, \quad 4-5. \quad -3, \quad 4-6. \quad 5, \quad 4-7. \quad -3 - 5i, \quad 4-8. \quad \sqrt{34}$

5. **Q.** $\alpha = 2 - 3i, \quad \beta = 5 - i$

A. 5-1. $7 - 4i, \quad 5-2. \quad -3 - 2i, \quad 5-3. \quad 7 - 17i, \quad 5-4. \quad \frac{1-i}{2}, \quad 5-5. \quad 5, \quad 5-6. \quad -1, \quad 5-7. \quad 5 + i, \quad 5-8. \quad \sqrt{26}$

6. **Q.** $\alpha = 2 + 5i, \quad \beta = -2 + i$

A. 6-1. $6i, \quad 6-2. \quad 4 + 4i, \quad 6-3. \quad -9 - 8i, \quad 6-4. \quad \frac{1-12i}{5}, \quad 6-5. \quad -2, \quad 6-6. \quad 1, \quad 6-7. \quad -2 - i, \quad 6-8. \quad \sqrt{5}$

7. **Q.** $\alpha = 5 + 4i, \quad \beta = 3 + i$

A. 7-1. $8 + 5i, \quad 7-2. \quad 2 + 3i, \quad 7-3. \quad 11 + 17i, \quad 7-4. \quad \frac{19+7i}{10}, \quad 7-5. \quad 3, \quad 7-6. \quad 1, \quad 7-7. \quad 3 - i, \quad 7-8. \quad \sqrt{10}$