代数学幾何学 (A/B) 計算演習 [問題] (2009/07/09)

問. 次の二つの複素ベクトル u, v の内積 (u, v) を求めなさい

Q.1

$$\mathrm{u} = \begin{pmatrix} 1-3i \\ 3-3i \\ 3+3i \\ 3+2i \\ 1-i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} 1+2i \\ -3+2i \\ 0 \\ 3-i \\ -1-2i \end{pmatrix}$$

Q.2

$$\mathrm{u} = \begin{pmatrix} -2+i \\ 2i \\ 3+i \\ -2+2i \\ 1-2i \\ -1+2i \\ 2-2i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} 1-3i \\ -1+2i \\ -1+2i \\ -i \\ -3 \\ -2+3i \\ 1+i \end{pmatrix}$$

Q.3

$$\mathrm{u} = \begin{pmatrix} -2 \\ -3+2i \\ 1 \\ 2-3i \\ i \\ 3+3i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} 2+i \\ 3 \\ -1-2i \\ 3+2i \\ -3-3i \\ -1+3i \end{pmatrix}$$

Q.4

$$\mathrm{u} = \begin{pmatrix} 1+2i \\ -3-i \\ -2+i \\ 3+3i \\ -1-i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} -3+3i \\ 3-2i \\ 1-2i \\ 1-3i \\ 2+3i \end{pmatrix}$$

Q.5

$$\mathrm{u} = \begin{pmatrix} 0 \\ -3+i \\ 1+3i \\ 2+2i \\ -1+i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} -3+i \\ 3+i \\ i \\ -3i \\ -1 \end{pmatrix}$$

Q.6

$$\mathrm{u} = \begin{pmatrix} -3-3i \\ -3-2i \\ -1+2i \\ 3-3i \\ 3+i \\ 3+2i \\ -2+i \end{pmatrix}, \mathrm{v} = \begin{pmatrix} 3+3i \\ -2 \\ -1-3i \\ -3 \\ 3i \\ 2-3i \\ -3-i \end{pmatrix}$$

代数学幾何学 (A/B) 計算演習 [解答] (2009/07/09)

A.1

$$
\left(\begin{pmatrix} 1-3i \\ 3-3i \\ 3+3i \\ 3+2i \\ 1-i \end{pmatrix}, \begin{pmatrix} 1+2i \\ -3+2i \\ 0 \\ 3-i \\ -1-2i \end{pmatrix}\right)
\begin{aligned}
=\ & (1-3i)\times\overline{(1+2i)}+(3-3i)\times\overline{(-3+2i)} \\
+\ & (3+3i)\times\overline{(0)}+(3+2i)\times\overline{(3-i)} \\
+\ & (1-i)\times\overline{(-1-2i)} \\
=\ & (1-3i)\times(1-2i)+(3-3i)\times(-3-2i) \\
+\ & (3+3i)\times(0)+(3+2i)\times(3+i) \\
+\ & (1-i)\times(-1+2i) \\
=\ & (-5-5i)+(-15+3i)+(0)+(7+9i) \\
+\ & (1+3i) \\
=\ & -12+10i
\end{aligned}
$$

A.2

$$
\left(\begin{pmatrix} -2+i \\ 2i \\ 3+i \\ -2+2i \\ 1-2i \\ -1+2i \\ 2-2i \end{pmatrix}, \begin{pmatrix} 1-3i \\ -1+2i \\ -1+2i \\ -i \\ -3 \\ -2+3i \\ 1+i \end{pmatrix}\right)
\begin{aligned}
=\ & (-2+i)\times\overline{(1-3i)}+(2i)\times\overline{(-1+2i)} \\
+\ & (3+i)\times\overline{(-1+2i)}+(-2+2i)\times\overline{(-i)} \\
+\ & (1-2i)\times\overline{(-3)}+(-1+2i)\times\overline{(-2+3i)} \\
+\ & (2-2i)\times\overline{(1+i)} \\
=\ & (-2+i)\times(1+3i)+(2i)\times(-1-2i) \\
+\ & (3+i)\times(-1-2i)+(-2+2i)\times(i) \\
+\ & (1-2i)\times(-3)+(-1+2i)\times(-2-3i) \\
+\ & (2-2i)\times(1-i) \\
=\ & (-5-5i)+(4-2i)+(-1-7i)+(-2-2i) \\
+\ & (-3+6i)+(8-i)+(-4i) \\
=\ & 1-15i
\end{aligned}
$$

A.3

$$\left(\begin{pmatrix} -2 \\ -3+2i \\ 1 \\ 2-3i \\ i \\ 3+3i \end{pmatrix}, \begin{pmatrix} 2+i \\ 3 \\ -1-2i \\ 3+2i \\ -3-3i \\ -1+3i \end{pmatrix}\right) = (-2)\times\overline{(2+i)} + (-3+2i)\times\overline{(3)}$$

$$\begin{aligned}
&+ (1)\times\overline{(-1-2i)} + (2-3i)\times\overline{(3+2i)} \\
&+ (i)\times\overline{(-3-3i)} + (3+3i)\times\overline{(-1+3i)} \\
=\ & (-2)\times(2-i) + (-3+2i)\times(3) \\
&+ (1)\times(-1+2i) + (2-3i)\times(3-2i) \\
&+ (i)\times(-3+3i) + (3+3i)\times(-1-3i) \\
=\ & (-4+2i) + (-9+6i) + (-1+2i) + (-13i) \\
&+ (-3-3i) + (6-12i) \\
=\ & -11-18i
\end{aligned}$$

A.4

$$\left(\begin{pmatrix} 1+2i \\ -3-i \\ -2+i \\ 3+3i \\ -1-i \end{pmatrix}, \begin{pmatrix} -3+3i \\ 3-2i \\ 1-2i \\ 1-3i \\ 2+3i \end{pmatrix}\right) = (1+2i)\times\overline{(-3+3i)} + (-3-i)\times\overline{(3-2i)}$$

$$\begin{aligned}
&+ (-2+i)\times\overline{(1-2i)} + (3+3i)\times\overline{(1-3i)} \\
&+ (-1-i)\times\overline{(2+3i)} \\
=\ & (1+2i)\times(-3-3i) + (-3-i)\times(3+2i) \\
&+ (-2+i)\times(1+2i) + (3+3i)\times(1+3i) \\
&+ (-1-i)\times(2-3i) \\
=\ & (3-9i) + (-7-9i) + (-4-3i) + (-6+12i) \\
&+ (-5+i) \\
=\ & -19-8i
\end{aligned}$$

A.5

$$
\begin{aligned}
\left(\begin{pmatrix} 0 \\ -3+i \\ 1+3i \\ 2+2i \\ -1+i \end{pmatrix}, \begin{pmatrix} -3+i \\ 3+i \\ i \\ -3i \\ -1 \end{pmatrix}\right) &= (0) \times \overline{(-3+i)} + (-3+i) \times \overline{(3+i)} \\
&+ (1+3i) \times \overline{(i)} + (2+2i) \times \overline{(-3i)} \\
&+ (-1+i) \times \overline{(-1)} \\
&= (0) \times (-3-i) + (-3+i) \times (3-i) \\
&+ (1+3i) \times (-i) + (2+2i) \times (3i) \\
&+ (-1+i) \times (-1) \\
&= (0) + (-8+6i) + (3-i) + (-6+6i) \\
&+ (1-i) \\
&= -10+10i
\end{aligned}
$$

A.6

$$
\begin{aligned}
\left(\begin{pmatrix} -3-3i \\ -3-2i \\ -1+2i \\ 3-3i \\ 3+i \\ 3+2i \\ -2+i \end{pmatrix}, \begin{pmatrix} 3+3i \\ -2 \\ -1-3i \\ -3 \\ 3i \\ 2-3i \\ -3-i \end{pmatrix}\right) &= (-3-3i) \times \overline{(3+3i)} + (-3-2i) \times \overline{(-2)} \\
&+ (-1+2i) \times \overline{(-1-3i)} + (3-3i) \times \overline{(-3)} \\
&+ (3+i) \times \overline{(3i)} + (3+2i) \times \overline{(2-3i)} \\
&+ (-2+i) \times \overline{(-3-i)} \\
&= (-3-3i) \times (3-3i) + (-3-2i) \times (-2) \\
&+ (-1+2i) \times (-1+3i) + (3-3i) \times (-3) \\
&+ (3+i) \times (-3i) + (3+2i) \times (2+3i) \\
&+ (-2+i) \times (-3+i) \\
&= (-18) + (6+4i) + (-5-5i) + (-9+9i) \\
&+ (3-9i) + (13i) + (5-5i) \\
&= -18+7i
\end{aligned}
$$