

代数学幾何学 (A/B) 計算演習 [問題] (2009/10/22)

問. 次の行列の行列式を求めなさい

Q.1

$$\begin{vmatrix} 0 & -2 & 4 & -3 \\ 0 & 7 & 0 & 1 \\ 4 & -3 & -9 & 7 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

Q.6

$$\begin{vmatrix} 0 & 1 & 7 & -7 & 3 \\ 1 & 0 & 4 & 6 & 6 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & -8 & 0 & 2 & 0 \\ 0 & -7 & 1 & 2 & 0 \end{vmatrix}$$

Q.2

$$\begin{vmatrix} 5 & 3 & 0 & 6 \\ -5 & 3 & 3 & -2 \\ 0 & 0 & 0 & 4 \\ 2 & 0 & 0 & -3 \end{vmatrix}$$

Q.7

$$\begin{vmatrix} -6 & 0 & 5 & 4 & -1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ -8 & 4 & 8 & 2 & 7 & -8 \\ 2 & 0 & 4 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 4 & 0 & -7 & -7 & 9 & 2 \end{vmatrix}$$

Q.3

$$\begin{vmatrix} -1 & 0 & 0 & 0 & 2 \\ -6 & 3 & 0 & 0 & 5 \\ 0 & 2 & 2 & 0 & 3 \\ -6 & 1 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Q.8

$$\begin{vmatrix} 8 & 5 & 2 & 0 & 0 \\ -8 & -1 & 8 & 2 & 4 \\ -8 & 3 & 8 & 0 & 2 \\ 4 & 0 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 & 0 \end{vmatrix}$$

Q.4

$$\begin{vmatrix} 0 & 8 & 6 & 1 & -4 & -8 \\ 0 & -2 & -7 & 0 & -9 & 1 \\ 1 & -9 & 4 & 6 & 3 & -6 \\ 0 & 2 & -6 & 0 & 0 & 0 \\ 0 & 7 & 2 & 0 & 2 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \end{vmatrix}$$

Q.9

$$\begin{vmatrix} 0 & 0 & -3 & 0 & 3 \\ 0 & 5 & -2 & 1 & -4 \\ 0 & 2 & -6 & 0 & 3 \\ 1 & -5 & -3 & -1 & 9 \\ 0 & 0 & 2 & 0 & 0 \end{vmatrix}$$

Q.5

$$\begin{vmatrix} 6 & 8 & -5 & 4 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{vmatrix}$$

Q.10

$$\begin{vmatrix} 2 & 0 & -4 & 0 \\ 0 & 0 & 4 & 0 \\ -3 & 3 & -1 & 0 \\ -3 & -4 & -1 & 2 \end{vmatrix}$$

代数学幾何学 (A/B) 計算演習 [解答] (2009/10/22)

A.1

$$\begin{aligned}
 \begin{vmatrix} 0 & -2 & 4 & -3 \\ 0 & 7 & 0 & 1 \\ 4 & -3 & -9 & 7 \\ 0 & 1 & 0 & 0 \end{vmatrix} &= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 7 & 0 & 0 & 1 \\ -3 & 4 & -9 & 7 \\ -2 & 0 & 4 & -3 \end{vmatrix} && \begin{array}{l} \text{1 行目と 4 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 &= \begin{vmatrix} 0 & 0 & 1 \\ 4 & -9 & 7 \\ 0 & 4 & -3 \end{vmatrix} \\
 &= - \begin{vmatrix} 1 & 0 & 0 \\ 7 & -9 & 4 \\ -3 & 4 & 0 \end{vmatrix} && \text{1 列目と 3 列目を交換} \\
 &= - \begin{vmatrix} -9 & 4 \\ 4 & 0 \end{vmatrix} \\
 &= \begin{vmatrix} 4 & 0 \\ -9 & 4 \end{vmatrix} && \text{1 行目と 2 行目を交換} \\
 &= 4 \mid 4 \mid \\
 &= 16
 \end{aligned}$$

A.2

$$\begin{aligned}
 \begin{vmatrix} 5 & 3 & 0 & 6 \\ -5 & 3 & 3 & -2 \\ 0 & 0 & 0 & 4 \\ 2 & 0 & 0 & -3 \end{vmatrix} &= \begin{vmatrix} 4 & 0 & 0 & 0 \\ -2 & 3 & 3 & -5 \\ 6 & 3 & 0 & 5 \\ -3 & 0 & 0 & 2 \end{vmatrix} && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 4 列目を交換} \end{array} \\
 &= 4 \begin{vmatrix} 3 & 3 & -5 \\ 3 & 0 & 5 \\ 0 & 0 & 2 \end{vmatrix} \\
 &= 4 \begin{vmatrix} 2 & 0 & 0 \\ 5 & 0 & 3 \\ -5 & 3 & 3 \end{vmatrix} && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
 &= 8 \begin{vmatrix} 0 & 3 \\ 3 & 3 \end{vmatrix} \\
 &= -8 \begin{vmatrix} 3 & 0 \\ 3 & 3 \end{vmatrix} && \text{1 列目と 2 列目を交換} \\
 &= -24 \mid 3 \mid \\
 &= -72
 \end{aligned}$$

A.3

$$\begin{aligned}
\begin{vmatrix} -1 & 0 & 0 & 0 & 2 \\ -6 & 3 & 0 & 0 & 5 \\ 0 & 2 & 2 & 0 & 3 \\ -6 & 1 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{vmatrix} &= - \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ -6 & 3 & 0 & 0 & 5 \\ 0 & 2 & 2 & 0 & 3 \\ -6 & 1 & 0 & 2 & 0 \\ -1 & 0 & 0 & 0 & 2 \end{vmatrix} && \text{1 行目と 5 行目を交換} \\
&= -2 \begin{vmatrix} 3 & 0 & 0 & 5 \\ 2 & 2 & 0 & 3 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix} \\
&= -2 \begin{vmatrix} 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 5 & 0 & 0 & 3 \end{vmatrix} && \begin{array}{l} \text{1 行目と 4 行目を交換} \\ \text{1 列目と 4 列目を交換} \end{array} \\
&= -4 \begin{vmatrix} 2 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} \\
&= -4 \begin{vmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 0 & 2 \end{vmatrix} && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
&= -12 \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \\
&= -24 \begin{vmatrix} 2 \end{vmatrix} \\
&= -48
\end{aligned}$$

A.4

$$\begin{aligned}
& \left| \begin{array}{cccccc} 0 & 8 & 6 & 1 & -4 & -8 \\ 0 & -2 & -7 & 0 & -9 & 1 \\ 1 & -9 & 4 & 6 & 3 & -6 \\ 0 & 2 & -6 & 0 & 0 & 0 \\ 0 & 7 & 2 & 0 & 2 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \end{array} \right| = \left| \begin{array}{cccccc} 3 & 0 & 0 & 0 & 0 & 0 \\ -7 & -2 & 0 & 0 & -9 & 1 \\ 4 & -9 & 1 & 6 & 3 & -6 \\ -6 & 2 & 0 & 0 & 0 & 0 \\ 2 & 7 & 0 & 0 & 2 & 0 \\ 6 & 8 & 0 & 1 & -4 & -8 \end{array} \right| \begin{array}{l} \text{1 行目と 6 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
& = 3 \left| \begin{array}{ccccc} -2 & 0 & 0 & -9 & 1 \\ -9 & 1 & 6 & 3 & -6 \\ 2 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 2 & 0 \\ 8 & 0 & 1 & -4 & -8 \end{array} \right| \\
& = -3 \left| \begin{array}{ccccc} 2 & 0 & 0 & 0 & 0 \\ -9 & 1 & 6 & 3 & -6 \\ -2 & 0 & 0 & -9 & 1 \\ 7 & 0 & 0 & 2 & 0 \\ 8 & 0 & 1 & -4 & -8 \end{array} \right| \begin{array}{l} \text{1 行目と 3 行目を交換} \end{array} \\
& = -6 \left| \begin{array}{cccc} 1 & 6 & 3 & -6 \\ 0 & 0 & -9 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & -4 & -8 \end{array} \right| \\
& = -6 \left| \begin{array}{cccc} 2 & 0 & 0 & 0 \\ -9 & 0 & 0 & 1 \\ 3 & 6 & 1 & -6 \\ -4 & 1 & 0 & -8 \end{array} \right| \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
& = -12 \left| \begin{array}{ccc} 0 & 0 & 1 \\ 6 & 1 & -6 \\ 1 & 0 & -8 \end{array} \right| \\
& = 12 \left| \begin{array}{ccc} 1 & 0 & 0 \\ -6 & 1 & 6 \\ -8 & 0 & 1 \end{array} \right| \begin{array}{l} \text{1 列目と 3 列目を交換} \end{array} \\
& = 12 \left| \begin{array}{cc} 1 & 6 \\ 0 & 1 \end{array} \right| \\
& = 12 \left| \begin{array}{cc} 1 & 0 \\ 6 & 1 \end{array} \right| \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
& = 12 \left| 1 \right| \\
& = 12
\end{aligned}$$

A.5

$$\begin{aligned}
 \left| \begin{array}{cccc} 6 & 8 & -5 & 4 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array} \right| &= \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 8 & 1 & 0 & 0 \\ -5 & 8 & 6 & 4 \\ 0 & 6 & 3 & 0 \end{array} \right| && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
 &= \left| \begin{array}{ccc} 1 & 0 & 0 \\ 8 & 6 & 4 \\ 6 & 3 & 0 \end{array} \right| \\
 &= \left| \begin{array}{cc} 6 & 4 \\ 3 & 0 \end{array} \right| \\
 &= - \left| \begin{array}{cc} 3 & 0 \\ 6 & 4 \end{array} \right| && \text{1 行目と 2 行目を交換} \\
 &= -3 \left| \begin{array}{c} 4 \end{array} \right| \\
 &= -12
 \end{aligned}$$

A.6

$$\begin{aligned}
 \left| \begin{array}{ccccc} 0 & 1 & 7 & -7 & 3 \\ 1 & 0 & 4 & 6 & 6 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & -8 & 0 & 2 & 0 \\ 0 & -7 & 1 & 2 & 0 \end{array} \right| &= \left| \begin{array}{ccccc} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 6 \\ 1 & 0 & 7 & -7 & 3 \\ -8 & 0 & 0 & 2 & 0 \\ -7 & 0 & 1 & 2 & 0 \end{array} \right| && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 &= 2 \left| \begin{array}{cccc} 1 & 4 & 6 & 6 \\ 0 & 7 & -7 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right| \\
 &= 2 \left| \begin{array}{cccc} 2 & 0 & 0 & 0 \\ -7 & 7 & 0 & 3 \\ 6 & 4 & 1 & 6 \\ 2 & 1 & 0 & 0 \end{array} \right| && \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
 &= 4 \left| \begin{array}{ccc} 7 & 0 & 3 \\ 4 & 1 & 6 \\ 1 & 0 & 0 \end{array} \right| \\
 &= -4 \left| \begin{array}{ccc} 1 & 0 & 0 \\ 4 & 1 & 6 \\ 7 & 0 & 3 \end{array} \right| && \text{1 行目と 3 行目を交換} \\
 &= -4 \left| \begin{array}{cc} 1 & 6 \\ 0 & 3 \end{array} \right| \\
 &= -4 \left| \begin{array}{cc} 3 & 0 \\ 6 & 1 \end{array} \right| && \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 &= -12 \left| \begin{array}{c} 1 \end{array} \right| \\
 &= -12
 \end{aligned}$$

A.7

$$\begin{aligned}
 & \begin{vmatrix} -6 & 0 & 5 & 4 & -1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ -8 & 4 & 8 & 2 & 7 & -8 \\ 2 & 0 & 4 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 4 & 0 & -7 & -7 & 9 & 2 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ -6 & 0 & 5 & 4 & -1 & 0 \\ -8 & 4 & 8 & 2 & 7 & -8 \\ 2 & 0 & 4 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 4 & 0 & -7 & -7 & 9 & 2 \end{vmatrix} & \text{1 行目と 2 行目を交換} \\
 & = -2 \begin{vmatrix} 0 & 5 & 4 & -1 & 0 \\ 4 & 8 & 2 & 7 & -8 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -7 & -7 & 9 & 2 \end{vmatrix} \\
 & = -2 \begin{vmatrix} 4 & 0 & 0 & 0 & 0 \\ 8 & 4 & 2 & 7 & -8 \\ 5 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -7 & 0 & -7 & 9 & 2 \end{vmatrix} & \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 & = -8 \begin{vmatrix} 4 & 2 & 7 & -8 \\ 0 & 4 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -7 & 9 & 2 \end{vmatrix} \\
 & = -8 \begin{vmatrix} 1 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 7 & 2 & 4 & -8 \\ 9 & -7 & 0 & 2 \end{vmatrix} & \begin{array}{l} \text{1 行目と 3 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\
 & = -8 \begin{vmatrix} 4 & 0 & 0 \\ 2 & 4 & -8 \\ -7 & 0 & 2 \end{vmatrix} \\
 & = -32 \begin{vmatrix} 4 & -8 \\ 0 & 2 \end{vmatrix} \\
 & = -32 \begin{vmatrix} 2 & 0 \\ -8 & 4 \end{vmatrix} & \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 2 列目を交換} \end{array} \\
 & = -64 | 4 | \\
 & = -256
 \end{aligned}$$

A.8

$$\begin{aligned}
 \begin{vmatrix} 8 & 5 & 2 & 0 & 0 \\ -8 & -1 & 8 & 2 & 4 \\ -8 & 3 & 8 & 0 & 2 \\ 4 & 0 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 & 0 \end{vmatrix} &= - \begin{vmatrix} 4 & 0 & 0 & 0 & 0 \\ -8 & -1 & 8 & 2 & 4 \\ -8 & 3 & 8 & 0 & 2 \\ 8 & 5 & 2 & 0 & 0 \\ 4 & 4 & 0 & 0 & 0 \end{vmatrix} && \text{1 行目と 4 行目を交換} \\
 &= -4 \begin{vmatrix} -1 & 8 & 2 & 4 \\ 3 & 8 & 0 & 2 \\ 5 & 2 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{vmatrix} \\
 &= 4 \begin{vmatrix} 4 & 0 & 0 & 0 \\ 3 & 8 & 0 & 2 \\ 5 & 2 & 0 & 0 \\ -1 & 8 & 2 & 4 \end{vmatrix} && \text{1 行目と 4 行目を交換} \\
 &= 16 \begin{vmatrix} 8 & 0 & 2 \\ 2 & 0 & 0 \\ 8 & 2 & 4 \end{vmatrix} \\
 &= -16 \begin{vmatrix} 2 & 0 & 0 \\ 8 & 0 & 2 \\ 8 & 2 & 4 \end{vmatrix} && \text{1 行目と 2 行目を交換} \\
 &= -32 \begin{vmatrix} 0 & 2 \\ 2 & 4 \end{vmatrix} \\
 &= 32 \begin{vmatrix} 2 & 0 \\ 4 & 2 \end{vmatrix} && \text{1 列目と 2 列目を交換} \\
 &= 64 \begin{vmatrix} 2 \\ 2 \end{vmatrix} \\
 &= 128
 \end{aligned}$$

A.9

$$\begin{aligned}
& \begin{vmatrix} 0 & 0 & -3 & 0 & 3 \\ 0 & 5 & -2 & 1 & -4 \\ 0 & 2 & -6 & 0 & 3 \\ 1 & -5 & -3 & -1 & 9 \\ 0 & 0 & 2 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ -2 & 5 & 0 & 1 & -4 \\ -6 & 2 & 0 & 0 & 3 \\ -3 & -5 & 1 & -1 & 9 \\ -3 & 0 & 0 & 0 & 3 \end{vmatrix} & \begin{array}{l} 1 \text{ 行目と } 5 \text{ 行目を交換} \\ 1 \text{ 列目と } 3 \text{ 列目を交換} \end{array} \\
& = 2 \begin{vmatrix} 5 & 0 & 1 & -4 \\ 2 & 0 & 0 & 3 \\ -5 & 1 & -1 & 9 \\ 0 & 0 & 0 & 3 \end{vmatrix} \\
& = 2 \begin{vmatrix} 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 9 & 1 & -1 & -5 \\ -4 & 0 & 1 & 5 \end{vmatrix} & \begin{array}{l} 1 \text{ 行目と } 4 \text{ 行目を交換} \\ 1 \text{ 列目と } 4 \text{ 列目を交換} \end{array} \\
& = 6 \begin{vmatrix} 0 & 0 & 2 \\ 1 & -1 & -5 \\ 0 & 1 & 5 \end{vmatrix} \\
& = -6 \begin{vmatrix} 2 & 0 & 0 \\ -5 & -1 & 1 \\ 5 & 1 & 0 \end{vmatrix} & 1 \text{ 列目と } 3 \text{ 列目を交換} \\
& = -12 \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} \\
& = 12 \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} & 1 \text{ 行目と } 2 \text{ 行目を交換} \\
& = 12 \begin{vmatrix} 1 \end{vmatrix} \\
& = 12
\end{aligned}$$

A.10

$$\begin{aligned} \begin{vmatrix} 2 & 0 & -4 & 0 \\ 0 & 0 & 4 & 0 \\ -3 & 3 & -1 & 0 \\ -3 & -4 & -1 & 2 \end{vmatrix} &= \begin{vmatrix} 4 & 0 & 0 & 0 \\ -4 & 0 & 2 & 0 \\ -1 & 3 & -3 & 0 \\ -1 & -4 & -3 & 2 \end{vmatrix} & \begin{array}{l} \text{1 行目と 2 行目を交換} \\ \text{1 列目と 3 列目を交換} \end{array} \\ &= 4 \begin{vmatrix} 0 & 2 & 0 \\ 3 & -3 & 0 \\ -4 & -3 & 2 \end{vmatrix} \\ &= -4 \begin{vmatrix} 2 & 0 & 0 \\ -3 & 3 & 0 \\ -3 & -4 & 2 \end{vmatrix} & \begin{array}{l} \text{1 列目と 2 列目を交換} \end{array} \\ &= -8 \begin{vmatrix} 3 & 0 \\ -4 & 2 \end{vmatrix} \\ &= -24 \begin{vmatrix} 2 \end{vmatrix} \\ &= -48 \end{aligned}$$