

代数学幾何学 (A/B) 計算演習 [問題] (2009/10/22)

問. 次の行列の行列式を求めなさい

Q.1

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 7 & 3 & 5 & -7 & -3 & 8 \\ 49 & 9 & 25 & 49 & 9 & 64 \\ 343 & 27 & 125 & -343 & -27 & 512 \\ 2401 & 81 & 625 & 2401 & 81 & 4096 \\ 16807 & 243 & 3125 & -16807 & -243 & 32768 \end{vmatrix}$$

Q.6

$$\begin{vmatrix} 1 & -6 & 36 & -216 & 1296 & -7776 \\ 1 & 2 & 4 & 8 & 16 & 32 \\ 1 & -8 & 64 & -512 & 4096 & -32768 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 4 & -8 & 16 & -32 \end{vmatrix}$$

Q.2

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & -8 & -3 & 6 \\ 4 & 1 & 64 & 9 & 36 \\ -8 & -1 & -512 & -27 & 216 \\ 16 & 1 & 4096 & 81 & 1296 \end{vmatrix}$$

Q.7

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ -2 & -4 & -5 & 5 \\ 4 & 16 & 25 & 25 \\ -8 & -64 & -125 & 125 \end{vmatrix}$$

Q.3

$$\begin{vmatrix} 1 & -3 & 9 & -27 & 81 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & -2 & 4 & -8 & 16 \\ 1 & 6 & 36 & 216 & 1296 \\ 1 & 9 & 81 & 729 & 6561 \end{vmatrix}$$

Q.8

$$\begin{vmatrix} 1 & -2 & 4 & -8 \\ 1 & 4 & 16 & 64 \\ 1 & -1 & 1 & -1 \\ 1 & -3 & 9 & -27 \end{vmatrix}$$

Q.4

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 4 & -1 & -5 & -3 \\ 16 & 1 & 25 & 9 \\ 64 & -1 & -125 & -27 \end{vmatrix}$$

Q.9

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & 7 & 5 & -1 & -5 \\ 16 & 49 & 25 & 1 & 25 \\ -64 & 343 & 125 & -1 & -125 \\ 256 & 2401 & 625 & 1 & 625 \end{vmatrix}$$

Q.5

$$\begin{vmatrix} 1 & -3 & 9 & -27 & 81 & -243 \\ 1 & 7 & 49 & 343 & 2401 & 16807 \\ 1 & -5 & 25 & -125 & 625 & -3125 \\ 1 & 9 & 81 & 729 & 6561 & 59049 \\ 1 & -7 & 49 & -343 & 2401 & -16807 \\ 1 & -6 & 36 & -216 & 1296 & -7776 \end{vmatrix}$$

Q.10

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & -4 & 16 & -64 \\ 1 & 5 & 25 & 125 \end{vmatrix}$$

代数学幾何学 (A/B) 計算演習 [解答] (2009/10/22)

A.1

$$\begin{aligned}
 \text{与式} &= \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 7 & 3 & 5 & -7 & -3 & 8 \\ 49 & 9 & 25 & 49 & 9 & 64 \\ 343 & 27 & 125 & -343 & -27 & 512 \\ 2401 & 81 & 625 & 2401 & 81 & 4096 \\ 16807 & 243 & 3125 & -16807 & -243 & 32768 \end{vmatrix} \\
 &= \begin{vmatrix} 7^0 & 3^0 & 5^0 & (-7)^0 & (-3)^0 & 8^0 \\ 7^1 & 3^1 & 5^1 & (-7)^1 & (-3)^1 & 8^1 \\ 7^2 & 3^2 & 5^2 & (-7)^2 & (-3)^2 & 8^2 \\ 7^3 & 3^3 & 5^3 & (-7)^3 & (-3)^3 & 8^3 \\ 7^4 & 3^4 & 5^4 & (-7)^4 & (-3)^4 & 8^4 \\ 7^5 & 3^5 & 5^5 & (-7)^5 & (-3)^5 & 8^5 \end{vmatrix} \\
 &= (8 - (-3)) \times (8 - (-7)) \times (8 - 5) \times (8 - 3) \times (8 - 7) \\
 &\quad \times (-3 - (-7)) \times (-3 - 5) \times (-3 - 3) \times (-3 - 7) \\
 &\quad \times (-7 - 5) \times (-7 - 3) \times (-7 - 7) \\
 &\quad \times (5 - 3) \times (5 - 7) \\
 &\quad \times (3 - 7) \\
 &= -1115258880
 \end{aligned}$$

A.2

$$\begin{aligned}
 \text{与式} &= \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & -8 & -3 & 6 \\ 4 & 1 & 64 & 9 & 36 \\ -8 & -1 & -512 & -27 & 216 \\ 16 & 1 & 4096 & 81 & 1296 \end{vmatrix} \\
 &= \begin{vmatrix} (-2)^0 & (-1)^0 & (-8)^0 & (-3)^0 & 6^0 \\ (-2)^1 & (-1)^1 & (-8)^1 & (-3)^1 & 6^1 \\ (-2)^2 & (-1)^2 & (-8)^2 & (-3)^2 & 6^2 \\ (-2)^3 & (-1)^3 & (-8)^3 & (-3)^3 & 6^3 \\ (-2)^4 & (-1)^4 & (-8)^4 & (-3)^4 & 6^4 \end{vmatrix} \\
 &= (6 - (-3)) \times (6 - (-8)) \times (6 - (-1)) \times (6 - (-2)) \\
 &\quad \times (-3 - (-8)) \times (-3 - (-1)) \times (-3 - (-2)) \\
 &\quad \times (-8 - (-1)) \times (-8 - (-2)) \\
 &\quad \times (-1 - (-2)) \\
 &= 2963520
 \end{aligned}$$

A.3

$$\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & -3 & 9 & -27 & 81 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & -2 & 4 & -8 & 16 \\ 1 & 6 & 36 & 216 & 1296 \\ 1 & 9 & 81 & 729 & 6561 \end{vmatrix} \\
&= \begin{vmatrix} (-3)^0 & (-3)^1 & (-3)^2 & (-3)^3 & (-3)^4 \\ 0^0 & 0^1 & 0^2 & 0^3 & 0^4 \\ (-2)^0 & (-2)^1 & (-2)^2 & (-2)^3 & (-2)^4 \\ 6^0 & 6^1 & 6^2 & 6^3 & 6^4 \\ 9^0 & 9^1 & 9^2 & 9^3 & 9^4 \end{vmatrix} \\
&= (9-6) \times (9-(-2)) \times (9-0) \times (9-(-3)) \\
&\quad \times (6-(-2)) \times (6-0) \times (6-(-3)) \\
&\quad \times (-2-0) \times (-2-(-3)) \\
&\quad \times (0-(-3)) \\
&= -9237888
\end{aligned}$$

A.4

$$\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 4 & -1 & -5 & -3 \\ 16 & 1 & 25 & 9 \\ 64 & -1 & -125 & -27 \end{vmatrix} \\
&= \begin{vmatrix} 4^0 & (-1)^0 & (-5)^0 & (-3)^0 \\ 4^1 & (-1)^1 & (-5)^1 & (-3)^1 \\ 4^2 & (-1)^2 & (-5)^2 & (-3)^2 \\ 4^3 & (-1)^3 & (-5)^3 & (-3)^3 \end{vmatrix} \\
&= (-3-(-5)) \times (-3-(-1)) \times (-3-4) \\
&\quad \times (-5-(-1)) \times (-5-4) \\
&\quad \times (-1-4) \\
&= -5040
\end{aligned}$$

$$\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & -3 & 9 & -27 & 81 & -243 \\ 1 & 7 & 49 & 343 & 2401 & 16807 \\ 1 & -5 & 25 & -125 & 625 & -3125 \\ 1 & 9 & 81 & 729 & 6561 & 59049 \\ 1 & -7 & 49 & -343 & 2401 & -16807 \\ 1 & -6 & 36 & -216 & 1296 & -7776 \end{vmatrix} \\
&= \begin{vmatrix} (-3)^0 & (-3)^1 & (-3)^2 & (-3)^3 & (-3)^4 & (-3)^5 \\ 7^0 & 7^1 & 7^2 & 7^3 & 7^4 & 7^5 \\ (-5)^0 & (-5)^1 & (-5)^2 & (-5)^3 & (-5)^4 & (-5)^5 \\ 9^0 & 9^1 & 9^2 & 9^3 & 9^4 & 9^5 \\ (-7)^0 & (-7)^1 & (-7)^2 & (-7)^3 & (-7)^4 & (-7)^5 \\ (-6)^0 & (-6)^1 & (-6)^2 & (-6)^3 & (-6)^4 & (-6)^5 \end{vmatrix} \\
&= (-6 - (-7)) \times (-6 - 9) \times (-6 - (-5)) \times (-6 - 7) \times (-6 - (-3)) \\
&\quad \times (-7 - 9) \times (-7 - (-5)) \times (-7 - 7) \times (-7 - (-3)) \\
&\quad \times (9 - (-5)) \times (9 - 7) \times (9 - (-3)) \\
&\quad \times (-5 - 7) \times (-5 - (-3)) \\
&\quad \times (7 - (-3)) \\
&= -1362821120
\end{aligned}$$

A.6

$$\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & -6 & 36 & -216 & 1296 & -7776 \\ 1 & 2 & 4 & 8 & 16 & 32 \\ 1 & -8 & 64 & -512 & 4096 & -32768 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 4 & -8 & 16 & -32 \end{vmatrix} \\
&= \begin{vmatrix} (-6)^0 & (-6)^1 & (-6)^2 & (-6)^3 & (-6)^4 & (-6)^5 \\ 2^0 & 2^1 & 2^2 & 2^3 & 2^4 & 2^5 \\ (-8)^0 & (-8)^1 & (-8)^2 & (-8)^3 & (-8)^4 & (-8)^5 \\ 1^0 & 1^1 & 1^2 & 1^3 & 1^4 & 1^5 \\ 0^0 & 0^1 & 0^2 & 0^3 & 0^4 & 0^5 \\ (-2)^0 & (-2)^1 & (-2)^2 & (-2)^3 & (-2)^4 & (-2)^5 \end{vmatrix} \\
&= (-2-0) \times (-2-1) \times (-2-(-8)) \times (-2-2) \times (-2-(-6)) \\
&\times (0-1) \times (0-(-8)) \times (0-2) \times (0-(-6)) \\
&\times (1-(-8)) \times (1-2) \times (1-(-6)) \\
&\times (-8-2) \times (-8-(-6)) \\
&\times (2-(-6)) \\
&= 557383680
\end{aligned}$$

A.7

$$\begin{aligned}
\text{与式} &= \begin{vmatrix} 1 & 1 & 1 & 1 \\ -2 & -4 & -5 & 5 \\ 4 & 16 & 25 & 25 \\ -8 & -64 & -125 & 125 \end{vmatrix} \\
&= \begin{vmatrix} (-2)^0 & (-4)^0 & (-5)^0 & 5^0 \\ (-2)^1 & (-4)^1 & (-5)^1 & 5^1 \\ (-2)^2 & (-4)^2 & (-5)^2 & 5^2 \\ (-2)^3 & (-4)^3 & (-5)^3 & 5^3 \end{vmatrix} \\
&= (5-(-5)) \times (5-(-4)) \times (5-(-2)) \\
&\times (-5-(-4)) \times (-5-(-2)) \\
&\times (-4-(-2)) \\
&= -3780
\end{aligned}$$

A.8

$$\begin{aligned}
 \text{与式} &= \begin{vmatrix} 1 & -2 & 4 & -8 \\ 1 & 4 & 16 & 64 \\ 1 & -1 & 1 & -1 \\ 1 & -3 & 9 & -27 \end{vmatrix} \\
 &= \begin{vmatrix} (-2)^0 & (-2)^1 & (-2)^2 & (-2)^3 \\ 4^0 & 4^1 & 4^2 & 4^3 \\ (-1)^0 & (-1)^1 & (-1)^2 & (-1)^3 \\ (-3)^0 & (-3)^1 & (-3)^2 & (-3)^3 \end{vmatrix} \\
 &= (-3 - (-1)) \times (-3 - 4) \times (-3 - (-2)) \\
 &\quad \times (-1 - 4) \times (-1 - (-2)) \\
 &\quad \times (4 - (-2)) \\
 &= 420
 \end{aligned}$$

A.9

$$\begin{aligned}
 \text{与式} &= \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & 7 & 5 & -1 & -5 \\ 16 & 49 & 25 & 1 & 25 \\ -64 & 343 & 125 & -1 & -125 \\ 256 & 2401 & 625 & 1 & 625 \end{vmatrix} \\
 &= \begin{vmatrix} (-4)^0 & 7^0 & 5^0 & (-1)^0 & (-5)^0 \\ (-4)^1 & 7^1 & 5^1 & (-1)^1 & (-5)^1 \\ (-4)^2 & 7^2 & 5^2 & (-1)^2 & (-5)^2 \\ (-4)^3 & 7^3 & 5^3 & (-1)^3 & (-5)^3 \\ (-4)^4 & 7^4 & 5^4 & (-1)^4 & (-5)^4 \end{vmatrix} \\
 &= (-5 - (-1)) \times (-5 - 5) \times (-5 - 7) \times (-5 - (-4)) \\
 &\quad \times (-1 - 5) \times (-1 - 7) \times (-1 - (-4)) \\
 &\quad \times (5 - 7) \times (5 - (-4)) \\
 &\quad \times (7 - (-4)) \\
 &= -13685760
 \end{aligned}$$

A.10

$$\begin{aligned} \text{与式} &= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & -4 & 16 & -64 \\ 1 & 5 & 25 & 125 \end{vmatrix} \\ &= \begin{vmatrix} 0^0 & 0^1 & 0^2 & 0^3 \\ 1^0 & 1^1 & 1^2 & 1^3 \\ (-4)^0 & (-4)^1 & (-4)^2 & (-4)^3 \\ 5^0 & 5^1 & 5^2 & 5^3 \end{vmatrix} \\ &= (5 - (-4)) \times (5 - 1) \times (5 - 0) \\ &\quad \times (-4 - 1) \times (-4 - 0) \\ &\quad \times (1 - 0) \\ &= 3600 \end{aligned}$$