

# 代数学幾何学 (A/B) 計算演習 [問題] (2009/11/05)

問. 次の行列の行列式を求めなさい

Q.1

$$\begin{vmatrix} -1 & 2 & 0 & 0 \\ 1 & 2 & 2 & -1 \\ 2 & 1 & -2 & 2 \\ -2 & 0 & -1 & 2 \end{vmatrix}$$

Q.7

$$\begin{vmatrix} -1 & -1 & 1 & -2 \\ -2 & -1 & -2 & 1 \\ -2 & -2 & 2 & -1 \\ 2 & -1 & -2 & -1 \end{vmatrix}$$

Q.2

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 2 & 2 & 2 & -2 \\ 2 & -1 & 2 & 1 \\ 1 & -2 & 2 & -2 \end{vmatrix}$$

Q.8

$$\begin{vmatrix} -2 & 0 & -1 & 2 \\ -1 & 1 & -1 & 1 \\ 2 & -1 & 0 & 0 \\ -1 & 1 & 2 & 2 \end{vmatrix}$$

Q.3

$$\begin{vmatrix} 1 & 1 & 2 & 2 \\ -1 & -1 & -2 & 2 \\ -1 & 2 & -1 & -1 \\ -1 & -2 & 1 & 0 \end{vmatrix}$$

Q.9

$$\begin{vmatrix} 1 & -1 & -1 & -2 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & -1 & 0 \\ -2 & -1 & 1 & -2 \end{vmatrix}$$

Q.4

$$\begin{vmatrix} 2 & -1 & 1 & -1 \\ -1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 2 \\ -1 & -2 & 0 & 1 \end{vmatrix}$$

Q.10

$$\begin{vmatrix} 2 & 0 & 1 & 1 \\ 2 & 0 & 0 & -2 \\ 2 & 2 & 1 & 2 \\ -1 & -1 & 1 & 0 \end{vmatrix}$$

Q.5

$$\begin{vmatrix} 2 & 0 & -2 & 2 \\ 2 & 1 & -1 & -1 \\ -1 & 1 & -1 & -2 \\ -1 & 0 & 1 & 2 \end{vmatrix}$$

Q.11

$$\begin{vmatrix} -1 & 0 & -1 & 0 \\ 2 & 1 & 0 & -2 \\ -1 & 2 & 1 & 1 \\ -1 & -2 & 1 & -1 \end{vmatrix}$$

Q.6

$$\begin{vmatrix} 0 & -1 & 2 & 1 \\ 0 & 1 & 2 & -1 \\ -1 & 0 & 0 & -2 \\ 2 & 2 & -2 & 0 \end{vmatrix}$$

Q.12

$$\begin{vmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -2 \\ 2 & -2 & 2 & 2 \\ 0 & 2 & 0 & -2 \end{vmatrix}$$

# 代数学幾何学 (A/B) 計算演習 [解答] (2009/11/05)

A.1

$$\begin{aligned}
 \begin{vmatrix} -1 & 2 & 0 & 0 \\ 1 & 2 & 2 & -1 \\ 2 & 1 & -2 & 2 \\ -2 & 0 & -1 & 2 \end{vmatrix} &= 1 \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 4 & 2 & -1 \\ 2 & 1 & -2 & 2 \\ -2 & 0 & -1 & 2 \end{vmatrix} && \text{左 } R(2,1;1) ; 2 \text{ 行目に} \\
 & & & \text{1 行目を 1 倍して、加} \\
 & & & \text{える} \\
 &= 1 \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 4 & 2 & -1 \\ 0 & 5 & -2 & 2 \\ -2 & 0 & -1 & 2 \end{vmatrix} && \text{左 } R(3,1;2) ; 3 \text{ 行目に} \\
 & & & \text{1 行目を 2 倍して、加} \\
 & & & \text{える} \\
 &= 1 \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 4 & 2 & -1 \\ 0 & 5 & -2 & 2 \\ 0 & -4 & -1 & 2 \end{vmatrix} && \text{左 } R(4,1;-2) ; 4 \text{ 行目} \\
 & & & \text{に 1 行目を } -2 \text{ 倍して、} \\
 & & & \text{加える} \\
 &= -1 \begin{vmatrix} 4 & 2 & -1 \\ 5 & -2 & 2 \\ -4 & -1 & 2 \end{vmatrix} && \\
 &= -1 \begin{vmatrix} 4 & 2 & -1 \\ 0 & -\frac{9}{2} & \frac{13}{4} \\ -4 & -1 & 2 \end{vmatrix} && \text{左 } R(3,2;-\frac{5}{4}) ; 3 \text{ 行目} \\
 & & & \text{に 2 行目を } -\frac{5}{4} \text{ 倍して、} \\
 & & & \text{加える} \\
 &= -1 \begin{vmatrix} 4 & 2 & -1 \\ 0 & -\frac{9}{2} & \frac{13}{4} \\ 0 & 1 & 1 \end{vmatrix} && \text{左 } R(4,2;1) ; 4 \text{ 行目に} \\
 & & & \text{2 行目を 1 倍して、加} \\
 & & & \text{える} \\
 &= -4 \begin{vmatrix} -\frac{9}{2} & \frac{13}{4} \\ 1 & 1 \end{vmatrix} && \\
 &= -4 \times ((-\frac{9}{2}) \times 1 - \frac{13}{4} \times 1) && \\
 &= 31 && 
 \end{aligned}$$

## A.2

$$\begin{aligned}
& \left| \begin{array}{cccc} 1 & 2 & 1 & 0 \\ 2 & 2 & 2 & -2 \\ 2 & -1 & 2 & 1 \\ 1 & -2 & 2 & -2 \end{array} \right| = 1 \left| \begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & -2 & 0 & -2 \\ 2 & -1 & 2 & 1 \\ 1 & -2 & 2 & -2 \end{array} \right| & \begin{array}{l} \text{左 } R(2,1;-2) ; 2 \text{ 行目} \\ \text{に 1 行目を } -2 \text{ 倍して、} \\ \text{加える} \end{array} \\
& = 1 \left| \begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & -5 & 0 & 1 \\ 1 & -2 & 2 & -2 \end{array} \right| & \begin{array}{l} \text{左 } R(3,1;-2) ; 3 \text{ 行目} \\ \text{に 1 行目を } -2 \text{ 倍して、} \\ \text{加える} \end{array} \\
& = 1 \left| \begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & -5 & 0 & 1 \\ 0 & -4 & 1 & -2 \end{array} \right| & \begin{array}{l} \text{左 } R(4,1;-1) ; 4 \text{ 行目} \\ \text{に 1 行目を } -1 \text{ 倍して、} \\ \text{加える} \end{array} \\
& = 1 \left| \begin{array}{ccc} -2 & 0 & -2 \\ -5 & 0 & 1 \\ -4 & 1 & -2 \end{array} \right| & \\
& = 1 \left| \begin{array}{ccc} -2 & 0 & -2 \\ 0 & 0 & 6 \\ -4 & 1 & -2 \end{array} \right| & \begin{array}{l} \text{左 } R(3,2;-\frac{5}{2}) ; 3 \text{ 行目} \\ \text{に 2 行目を } -\frac{5}{2} \text{ 倍して、} \\ \text{加える} \end{array} \\
& = 1 \left| \begin{array}{ccc} -2 & 0 & -2 \\ 0 & 0 & 6 \\ 0 & 1 & 2 \end{array} \right| & \begin{array}{l} \text{左 } R(4,2;-2) ; 4 \text{ 行目} \\ \text{に 2 行目を } -2 \text{ 倍して、} \\ \text{加える} \end{array} \\
& = -2 \left| \begin{array}{cc} 0 & 6 \\ 1 & 2 \end{array} \right| \\
& = -2 \times (0 \times 2 - 6 \times 1) \\
& = 12
\end{aligned}$$

## A.3

$$\begin{aligned}
& \left| \begin{array}{cccc} 1 & 1 & 2 & 2 \\ -1 & -1 & -2 & 2 \\ -1 & 2 & -1 & -1 \\ -1 & -2 & 1 & 0 \end{array} \right| = 1 \left| \begin{array}{cccc} 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 4 \\ -1 & 2 & -1 & -1 \\ -1 & -2 & 1 & 0 \end{array} \right| & \text{左 } R(2,1;1) ; 2 \text{ 行目に} \\
& & & \text{1 行目を 1 倍して、加} \\
& & & \text{える} \\
& = 1 \left| \begin{array}{cccc} 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 4 \\ 0 & 3 & 1 & 1 \\ -1 & -2 & 1 & 0 \end{array} \right| & \text{左 } R(3,1;1) ; 3 \text{ 行目に} \\
& & & \text{1 行目を 1 倍して、加} \\
& & & \text{える} \\
& = 1 \left| \begin{array}{cccc} 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 4 \\ 0 & 3 & 1 & 1 \\ 0 & -1 & 3 & 2 \end{array} \right| & \text{左 } R(4,1;1) ; 4 \text{ 行目に} \\
& & & \text{1 行目を 1 倍して、加} \\
& & & \text{える} \\
& = 1 \left| \begin{array}{ccc} 0 & 0 & 4 \\ 3 & 1 & 1 \\ -1 & 3 & 2 \end{array} \right| \\
& = -1 \left| \begin{array}{ccc} 3 & 1 & 1 \\ 0 & 0 & 4 \\ -1 & 3 & 2 \end{array} \right| & \text{左 } P(3,2) ; 3 \text{ 行目と 2 行目を交換} \\
& = -1 \left| \begin{array}{ccc} 3 & 1 & 1 \\ 0 & 0 & 4 \\ 0 & \frac{10}{3} & \frac{7}{3} \end{array} \right| & \text{左 } R(4,2;\frac{1}{3}) ; 4 \text{ 行目に} \\
& & & \text{2 行目を } \frac{1}{3} \text{ 倍して、加} \\
& & & \text{える} \\
& = -3 \left| \begin{array}{cc} 0 & 4 \\ \frac{10}{3} & \frac{7}{3} \end{array} \right| \\
& = -3 \times (0 \times \frac{7}{3} - 4 \times \frac{10}{3}) \\
& = 40
\end{aligned}$$

A.4

$$\begin{aligned}
 & \left| \begin{array}{cccc} 2 & -1 & 1 & -1 \\ -1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 2 \\ -1 & -2 & 0 & 1 \end{array} \right| = 1 \left| \begin{array}{cccc} 2 & -1 & 1 & -1 \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 2 \\ -1 & -2 & 0 & 1 \end{array} \right| && \text{左 } R(2,1;\frac{1}{2}); 2 \text{ 行目に} \\
 & && \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\
 & && \text{える} \\
 & = 1 \left| \begin{array}{cccc} 2 & -1 & 1 & -1 \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{3}{2} & -\frac{1}{2} & \frac{5}{2} \\ -1 & -2 & 0 & 1 \end{array} \right| && \text{左 } R(3,1;-\frac{1}{2}); 3 \text{ 行目} \\
 & && \text{に 1 行目を } -\frac{1}{2} \text{ 倍して、} \\
 & && \text{加える} \\
 & = 1 \left| \begin{array}{cccc} 2 & -1 & 1 & -1 \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{3}{2} & -\frac{1}{2} & \frac{5}{2} \\ 0 & -\frac{5}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right| && \text{左 } R(4,1;\frac{1}{2}); 4 \text{ 行目に} \\
 & && \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\
 & && \text{える} \\
 & = 2 \left| \begin{array}{ccc} \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} & \frac{5}{2} \\ \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} \end{array} \right| && \\
 & = 2 \left| \begin{array}{ccc} \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & -1 & 2 \\ -\frac{5}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right| && \text{左 } R(3,2;-1); 3 \text{ 行目} \\
 & && \text{に 2 行目を } -1 \text{ 倍して、} \\
 & && \text{加える} \\
 & = 2 \left| \begin{array}{ccc} \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & -1 & 2 \\ 0 & \frac{4}{3} & \frac{4}{3} \end{array} \right| && \text{左 } R(4,2;\frac{5}{3}); 4 \text{ 行目に} \\
 & && \text{2 行目を } \frac{5}{3} \text{ 倍して、加} \\
 & && \text{える} \\
 & = 3 \left| \begin{array}{cc} -1 & 2 \\ \frac{4}{3} & \frac{4}{3} \end{array} \right| \\
 & = 3 \times ((-1) \times \frac{4}{3} - 2 \times \frac{4}{3}) \\
 & = -12
 \end{aligned}$$

A.5

$$\begin{aligned}
 & \begin{vmatrix} 2 & 0 & -2 & 2 \\ 2 & 1 & -1 & -1 \\ -1 & 1 & -1 & -2 \\ -1 & 0 & 1 & 2 \end{vmatrix} = 1 \begin{vmatrix} 2 & 0 & -2 & 2 \\ 0 & 1 & 1 & -3 \\ -1 & 1 & -1 & -2 \\ -1 & 0 & 1 & 2 \end{vmatrix} && \begin{array}{l} \text{左 } R(2,1;-1) ; 2 \text{ 行目} \\ \text{に } 1 \text{ 行目を } -1 \text{ 倍して、} \\ \text{加える} \end{array} \\
 & = 1 \begin{vmatrix} 2 & 0 & -2 & 2 \\ 0 & 1 & 1 & -3 \\ 0 & 1 & -2 & -1 \\ -1 & 0 & 1 & 2 \end{vmatrix} && \begin{array}{l} \text{左 } R(3,1;\frac{1}{2}) ; 3 \text{ 行目に} \\ 1 \text{ 行目を } \frac{1}{2} \text{ 倍して、加} \\ \text{える} \end{array} \\
 & = 1 \begin{vmatrix} 2 & 0 & -2 & 2 \\ 0 & 1 & 1 & -3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 3 \end{vmatrix} && \begin{array}{l} \text{左 } R(4,1;\frac{1}{2}) ; 4 \text{ 行目に} \\ 1 \text{ 行目を } \frac{1}{2} \text{ 倍して、加} \\ \text{える} \end{array} \\
 & = 2 \begin{vmatrix} 1 & 1 & -3 \\ 1 & -2 & -1 \\ 0 & 0 & 3 \end{vmatrix} \\
 & = 2 \begin{vmatrix} 1 & 1 & -3 \\ 0 & -3 & 2 \\ 0 & 0 & 3 \end{vmatrix} && \begin{array}{l} \text{左 } R(3,2;-1) ; 3 \text{ 行目} \\ \text{に } 2 \text{ 行目を } -1 \text{ 倍して、} \\ \text{加える} \end{array} \\
 & = 2 \begin{vmatrix} -3 & 2 \\ 0 & 3 \end{vmatrix} \\
 & = 2 \times ((-3) \times 3 - 2 \times 0) \\
 & = -18
 \end{aligned}$$

## A.6

$$\begin{aligned}
 \begin{vmatrix} 0 & -1 & 2 & 1 \\ 0 & 1 & 2 & -1 \\ -1 & 0 & 0 & -2 \\ 2 & 2 & -2 & 0 \end{vmatrix} &= -1 \begin{vmatrix} -1 & 0 & 0 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & 2 & 1 \\ 2 & 2 & -2 & 0 \end{vmatrix} && \text{左 P(3,1) ; 3 行目と 1 行目を交換} \\
 &= -1 \begin{vmatrix} -1 & 0 & 0 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & 2 & 1 \\ 0 & 2 & -2 & -4 \end{vmatrix} && \text{左 R(4,1;2) ; 4 行目に} \\
 & && \text{1 行目を 2 倍して、加} \\
 & && \text{える} \\
 &= 1 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 1 \\ 2 & -2 & -4 \end{vmatrix} \\
 &= 1 \begin{vmatrix} 1 & 2 & -1 \\ 0 & 4 & 0 \\ 2 & -2 & -4 \end{vmatrix} && \text{左 R(3,2;1) ; 3 行目に} \\
 & && \text{2 行目を 1 倍して、加} \\
 & && \text{える} \\
 &= 1 \begin{vmatrix} 1 & 2 & -1 \\ 0 & 4 & 0 \\ 0 & -6 & -2 \end{vmatrix} && \text{左 R(4,2;-2) ; 4 行目} \\
 & && \text{に 2 行目を -2 倍して、} \\
 & && \text{加える} \\
 &= 1 \begin{vmatrix} 4 & 0 \\ -6 & -2 \end{vmatrix} \\
 &= 1 \times (4 \times (-2) - 0 \times (-6)) \\
 &= -8
 \end{aligned}$$

## A.7

$$\begin{aligned}
& \begin{vmatrix} -1 & -1 & 1 & -2 \\ -2 & -1 & -2 & 1 \\ -2 & -2 & 2 & -1 \\ 2 & -1 & -2 & -1 \end{vmatrix} = 1 \begin{vmatrix} -1 & -1 & 1 & -2 \\ 0 & 1 & -4 & 5 \\ -2 & -2 & 2 & -1 \\ 2 & -1 & -2 & -1 \end{vmatrix} && \text{左 } R(2,1;-2) ; 2 \text{ 行目} \\
& && \text{に 1 行目を } -2 \text{ 倍して、} \\
& && \text{加える} \\
& = 1 \begin{vmatrix} -1 & -1 & 1 & -2 \\ 0 & 1 & -4 & 5 \\ 0 & 0 & 0 & 3 \\ 2 & -1 & -2 & -1 \end{vmatrix} && \text{左 } R(3,1;-2) ; 3 \text{ 行目} \\
& && \text{に 1 行目を } -2 \text{ 倍して、} \\
& && \text{加える} \\
& = 1 \begin{vmatrix} -1 & -1 & 1 & -2 \\ 0 & 1 & -4 & 5 \\ 0 & 0 & 0 & 3 \\ 0 & -3 & 0 & -5 \end{vmatrix} && \text{左 } R(4,1;2) ; 4 \text{ 行目に} \\
& && \text{1 行目を } 2 \text{ 倍して、加} \\
& && \text{える} \\
& = -1 \begin{vmatrix} 1 & -4 & 5 \\ 0 & 0 & 3 \\ -3 & 0 & -5 \end{vmatrix} \\
& = -1 \begin{vmatrix} 1 & -4 & 5 \\ 0 & 0 & 3 \\ 0 & -12 & 10 \end{vmatrix} && \text{左 } R(4,2;3) ; 4 \text{ 行目に} \\
& && \text{2 行目を } 3 \text{ 倍して、加} \\
& && \text{える} \\
& = -1 \begin{vmatrix} 0 & 3 \\ -12 & 10 \end{vmatrix} \\
& = -1 \times (0 \times 10 - 3 \times (-12)) \\
& = -36
\end{aligned}$$



$$\begin{aligned}
& \begin{vmatrix} -2 & 0 & -1 & 2 \\ -1 & 1 & -1 & 1 \\ 2 & -1 & 0 & 0 \\ -1 & 1 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} -2 & 0 & -1 & 2 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 2 & -1 & 0 & 0 \\ -1 & 1 & 2 & 2 \end{vmatrix} && \text{左 R(2,1;}-\frac{1}{2}) ; 2 \text{ 行目} \\
& && \text{に 1 行目を } -\frac{1}{2} \text{ 倍して、} \\
& && \text{加える} \\
& = 1 \begin{vmatrix} -2 & 0 & -1 & 2 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & -1 & -1 & 2 \\ -1 & 1 & 2 & 2 \end{vmatrix} && \text{左 R(3,1;1) ; 3 行目に} \\
& && \text{1 行目を 1 倍して、加} \\
& && \text{える} \\
& = 1 \begin{vmatrix} -2 & 0 & -1 & 2 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & -1 & -1 & 2 \\ 0 & 1 & \frac{5}{2} & 1 \end{vmatrix} && \text{左 R(4,1;}-\frac{1}{2}) ; 4 \text{ 行目} \\
& && \text{に 1 行目を } -\frac{1}{2} \text{ 倍して、} \\
& && \text{加える} \\
& = -2 \begin{vmatrix} 1 & -\frac{1}{2} & 0 \\ -1 & -1 & 2 \\ 1 & \frac{5}{2} & 1 \end{vmatrix} \\
& = -2 \begin{vmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & -\frac{3}{2} & 2 \\ 1 & \frac{5}{2} & 1 \end{vmatrix} && \text{左 R(3,2;1) ; 3 行目に} \\
& && \text{2 行目を 1 倍して、加} \\
& && \text{える} \\
& = -2 \begin{vmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & -\frac{3}{2} & 2 \\ 0 & 3 & 1 \end{vmatrix} && \text{左 R(4,2;-1) ; 4 行目} \\
& && \text{に 2 行目を } -1 \text{ 倍して、} \\
& && \text{加える} \\
& = -2 \begin{vmatrix} -\frac{3}{2} & 2 \\ 3 & 1 \end{vmatrix} \\
& = -2 \times ((-\frac{3}{2}) \times 1 - 2 \times 3) \\
& = 15
\end{aligned}$$

A.9

$$\begin{aligned} \left| \begin{array}{cccc} 1 & -1 & -1 & -2 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & -1 & 0 \\ -2 & -1 & 1 & -2 \end{array} \right| &= 1 \left| \begin{array}{cccc} 1 & -1 & -1 & -2 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & -1 & 0 \\ 0 & -3 & -1 & -6 \end{array} \right| && \text{左 } R(4,1;2) ; 4 \text{ 行目に} \\ &&& \text{1 行目を 2 倍して、加} \\ &&& \text{える} \\ &= 1 \left| \begin{array}{ccc} -1 & -1 & -2 \\ -1 & -1 & 0 \\ -3 & -1 & -6 \end{array} \right| \\ &= 1 \left| \begin{array}{ccc} -1 & -1 & -2 \\ 0 & 0 & 2 \\ -3 & -1 & -6 \end{array} \right| && \text{左 } R(3,2;-1) ; 3 \text{ 行目} \\ &&& \text{に 2 行目を } -1 \text{ 倍して、} \\ &&& \text{加える} \\ &= 1 \left| \begin{array}{ccc} -1 & -1 & -2 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{array} \right| && \text{左 } R(4,2;-3) ; 4 \text{ 行目} \\ &&& \text{に 2 行目を } -3 \text{ 倍して、} \\ &&& \text{加える} \\ &= -1 \left| \begin{array}{cc} 0 & 2 \\ 2 & 0 \end{array} \right| \\ &= -1 \times (0 \times 0 - 2 \times 2) \\ &= 4 \end{aligned}$$

A.10

$$\begin{aligned}
 \begin{vmatrix} 2 & 0 & 1 & 1 \\ 2 & 0 & 0 & -2 \\ 2 & 2 & 1 & 2 \\ -1 & -1 & 1 & 0 \end{vmatrix} &= 1 \begin{vmatrix} 2 & 0 & 1 & 1 \\ 0 & 0 & -1 & -3 \\ 2 & 2 & 1 & 2 \\ -1 & -1 & 1 & 0 \end{vmatrix} && \text{左 } R(2,1;-1) ; 2 \text{ 行目} \\
 & & & \text{に 1 行目を } -1 \text{ 倍して、} \\
 & & & \text{加える} \\
 &= 1 \begin{vmatrix} 2 & 0 & 1 & 1 \\ 0 & 0 & -1 & -3 \\ 0 & 2 & 0 & 1 \\ -1 & -1 & 1 & 0 \end{vmatrix} && \text{左 } R(3,1;-1) ; 3 \text{ 行目} \\
 & & & \text{に 1 行目を } -1 \text{ 倍して、} \\
 & & & \text{加える} \\
 &= 1 \begin{vmatrix} 2 & 0 & 1 & 1 \\ 0 & 0 & -1 & -3 \\ 0 & 2 & 0 & 1 \\ 0 & -1 & \frac{3}{2} & \frac{1}{2} \end{vmatrix} && \text{左 } R(4,1;\frac{1}{2}) ; 4 \text{ 行目に} \\
 & & & \text{1 行目を } \frac{1}{2} \text{ 倍して、加} \\
 & & & \text{える} \\
 &= 2 \begin{vmatrix} 0 & -1 & -3 \\ 2 & 0 & 1 \\ -1 & \frac{3}{2} & \frac{1}{2} \end{vmatrix} \\
 &= -2 \begin{vmatrix} 2 & 0 & 1 \\ 0 & -1 & -3 \\ -1 & \frac{3}{2} & \frac{1}{2} \end{vmatrix} && \text{左 } P(3,2) ; 3 \text{ 行目と } 2 \text{ 行目を交換} \\
 &= -2 \begin{vmatrix} 2 & 0 & 1 \\ 0 & -1 & -3 \\ 0 & \frac{3}{2} & 1 \end{vmatrix} && \text{左 } R(4,2;\frac{1}{2}) ; 4 \text{ 行目に} \\
 & & & \text{2 行目を } \frac{1}{2} \text{ 倍して、加} \\
 & & & \text{える} \\
 &= -4 \begin{vmatrix} -1 & -3 \\ \frac{3}{2} & 1 \end{vmatrix} \\
 &= -4 \times ((-1) \times 1 - (-3) \times \frac{3}{2}) \\
 &= -14
 \end{aligned}$$

A.11

$$\begin{aligned}
 & \begin{vmatrix} -1 & 0 & -1 & 0 \\ 2 & 1 & 0 & -2 \\ -1 & 2 & 1 & 1 \\ -1 & -2 & 1 & -1 \end{vmatrix} = 1 \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & -2 \\ -1 & 2 & 1 & 1 \\ -1 & -2 & 1 & -1 \end{vmatrix} && \text{左 R(2,1;2) ; 2行目に} \\
 & && \text{1行目を2倍して、加} \\
 & && \text{える} \\
 & = 1 \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & 2 & 2 & 1 \\ -1 & -2 & 1 & -1 \end{vmatrix} && \text{左 R(3,1;-1) ; 3行目} \\
 & && \text{に1行目を-1倍して、} \\
 & && \text{加える} \\
 & = 1 \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & 2 & 2 & 1 \\ 0 & -2 & 2 & -1 \end{vmatrix} && \text{左 R(4,1;-1) ; 4行目} \\
 & && \text{に1行目を-1倍して、} \\
 & && \text{加える} \\
 & = -1 \begin{vmatrix} 1 & -2 & -2 \\ 2 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix} \\
 & = -1 \begin{vmatrix} 1 & -2 & -2 \\ 0 & 6 & 5 \\ -2 & 2 & -1 \end{vmatrix} && \text{左 R(3,2;-2) ; 3行目} \\
 & && \text{に2行目を-2倍して、} \\
 & && \text{加える} \\
 & = -1 \begin{vmatrix} 1 & -2 & -2 \\ 0 & 6 & 5 \\ 0 & -2 & -5 \end{vmatrix} && \text{左 R(4,2;2) ; 4行目に} \\
 & && \text{2行目を2倍して、加} \\
 & && \text{える} \\
 & = -1 \begin{vmatrix} 6 & 5 \\ -2 & -5 \end{vmatrix} \\
 & = -1 \times (6 \times (-5) - 5 \times (-2)) \\
 & = 20
 \end{aligned}$$

A.12

$$\begin{aligned} \begin{vmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -2 \\ 2 & -2 & 2 & 2 \\ 0 & 2 & 0 & -2 \end{vmatrix} &= 1 \begin{vmatrix} 1 & -1 & -1 & 1 \\ 0 & 0 & -2 & -1 \\ 2 & -2 & 2 & 2 \\ 0 & 2 & 0 & -2 \end{vmatrix} && \text{左 R(2,1;1) ; 2行目に} \\ & & & \text{1行目を1倍して、加} \\ & & & \text{える} \\ &= 1 \begin{vmatrix} 1 & -1 & -1 & 1 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 4 & 0 \\ 0 & 2 & 0 & -2 \end{vmatrix} && \text{左 R(3,1;-2) ; 3行目} \\ & & & \text{に1行目を-2倍して、} \\ & & & \text{加える} \\ &= 1 \begin{vmatrix} 0 & -2 & -1 \\ 0 & 4 & 0 \\ 2 & 0 & -2 \end{vmatrix} \\ &= -1 \begin{vmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ 0 & -2 & -1 \end{vmatrix} && \text{左 P(4,2) ; 4行目と2行目を交換} \\ &= -2 \begin{vmatrix} 4 & 0 \\ -2 & -1 \end{vmatrix} \\ &= -2 \times (4 \times (-1) - 0 \times (-2)) \\ &= 8 \end{aligned}$$