

# 代数学幾何学 (A/B) 計算演習 [問題] (2009/11/05)

問. 次の連立方程式をクラメールの公式を用いて解きなさい。

Q.1

$$\begin{cases} -4x_1 + x_2 + x_3 + 2x_4 = 9 \\ 3x_1 - x_2 - x_3 = -10 \\ -x_1 + x_2 = 6 \\ -x_2 + x_3 - x_4 = -3 \end{cases}$$

Q.8

$$\begin{cases} -7x_1 + 5x_2 + 3x_3 = 32 \\ 10x_1 - 10x_2 - 3x_3 = -53 \\ 9x_1 - 6x_2 - 4x_3 = -40 \end{cases}$$

Q.2

$$\begin{cases} 2x_1 - x_3 = -7 \\ -3x_1 + 2x_2 = 16 \\ x_1 - 3x_2 + 2x_3 = -12 \end{cases}$$

Q.9

$$\begin{cases} x_1 + 4x_2 - 3x_3 = -29 \\ x_1 - 3x_2 = 8 \\ -x_1 - x_3 + x_4 = 2 \\ -x_1 + x_2 + x_3 = 3 \end{cases}$$

Q.3

$$\begin{cases} -x_1 + 5x_3 - x_4 = 0 \\ 2x_1 + x_2 + x_3 - 2x_4 = 2 \\ -2x_1 - x_2 - 4x_3 + 3x_4 = -2 \\ -2x_2 + x_4 = -5 \end{cases}$$

Q.10

$$\begin{cases} 8x_1 + 14x_2 - 13x_3 = -33 \\ -13x_1 - 22x_2 + 21x_3 = 54 \\ 2x_1 + 3x_2 - 3x_3 = -8 \end{cases}$$

Q.4

$$\begin{cases} 17x_1 - 16x_2 + 4x_3 = 79 \\ -10x_1 + 10x_2 - 3x_3 = -47 \\ -4x_1 + 3x_2 = -18 \end{cases}$$

Q.11

$$\begin{cases} 3x_1 - 4x_2 + 2x_3 = 23 \\ -3x_1 + 4x_2 - x_3 = -20 \\ -2x_1 + 3x_2 - 2x_3 = -18 \end{cases}$$

Q.5

$$\begin{cases} 4x_1 + 7x_2 - 9x_3 = -11 \\ -3x_2 + 2x_3 = 8 \\ -3x_1 - 4x_2 + 6x_3 = 5 \end{cases}$$

Q.12

$$\begin{cases} -3x_1 + 3x_2 + x_3 = 14 \\ x_1 - x_2 = -5 \\ x_1 - 2x_3 = -1 \end{cases}$$

Q.6

$$\begin{cases} x_1 - 2x_2 = -8 \\ x_1 + 2x_2 - x_3 = 2 \\ -x_1 - x_2 + x_3 = 1 \end{cases}$$

Q.13

$$\begin{cases} -2x_1 + x_2 = 11 \\ -x_1 + 2x_2 - 5x_3 = -5 \\ 3x_1 - 2x_2 + 2x_3 = -12 \end{cases}$$

Q.7

$$\begin{cases} 2x_1 - x_2 - 2x_3 - x_4 = -1 \\ -3x_2 + 3x_3 + x_4 = 4 \\ -x_1 + x_2 + x_3 = 0 \\ -x_1 + x_2 + x_4 = -1 \end{cases}$$

Q.14

$$\begin{cases} 11x_1 - 9x_2 - 7x_3 + 4x_4 = 41 \\ -2x_1 + 2x_2 + x_3 - x_4 = -7 \\ x_1 + 2x_3 - 2x_4 = 0 \\ -3x_1 + 2x_2 + x_3 = -10 \end{cases}$$

# 代数学幾何学 (A/B) 計算演習 [解答] (2009/11/05)

A.1

$$|A| = \begin{vmatrix} -4 & 1 & 1 & 2 \\ 3 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{vmatrix} = -1$$

$$|A_1| = \begin{vmatrix} 9 & 1 & 1 & 2 \\ -10 & -1 & -1 & 0 \\ 6 & 1 & 0 & 0 \\ -3 & -1 & 1 & -1 \end{vmatrix} = 3$$

$$|A_2| = \begin{vmatrix} -4 & 9 & 1 & 2 \\ 3 & -10 & -1 & 0 \\ -1 & 6 & 0 & 0 \\ 0 & -3 & 1 & -1 \end{vmatrix} = -3$$

$$|A_3| = \begin{vmatrix} -4 & 1 & 9 & 2 \\ 3 & -1 & -10 & 0 \\ -1 & 1 & 6 & 0 \\ 0 & -1 & -3 & -1 \end{vmatrix} = 2$$

$$|A_4| = \begin{vmatrix} -4 & 1 & 1 & 9 \\ 3 & -1 & -1 & -10 \\ -1 & 1 & 0 & 6 \\ 0 & -1 & 1 & -3 \end{vmatrix} = 2$$

A.2

$$|A| = \begin{vmatrix} 2 & 0 & -1 \\ -3 & 2 & 0 \\ 1 & -3 & 2 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} -7 & 0 & -1 \\ 16 & 2 & 0 \\ -12 & -3 & 2 \end{vmatrix} = -4$$

$$|A_2| = \begin{vmatrix} 2 & -7 & -1 \\ -3 & 16 & 0 \\ 1 & -12 & 2 \end{vmatrix} = 2$$

$$|A_3| = \begin{vmatrix} 2 & 0 & -7 \\ -3 & 2 & 16 \\ 1 & -3 & -12 \end{vmatrix} = -1$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{3}{-1} = -3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-3}{-1} = 3 \\ x_3 = \frac{|A_3|}{|A|} = \frac{2}{-1} = -2 \\ x_4 = \frac{|A_4|}{|A|} = \frac{2}{-1} = -2 \end{cases}$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-4}{1} = -4 \\ x_2 = \frac{|A_2|}{|A|} = \frac{2}{1} = 2 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-1}{1} = -1 \end{cases}$$

A.3

$$|A| = \begin{vmatrix} -1 & 0 & 5 & -1 \\ 2 & 1 & 1 & -2 \\ -2 & -1 & -4 & 3 \\ 0 & -2 & 0 & 1 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} 0 & 0 & 5 & -1 \\ 2 & 1 & 1 & -2 \\ -2 & -1 & -4 & 3 \\ -5 & -2 & 0 & 1 \end{vmatrix} = -2$$

$$|A_2| = \begin{vmatrix} -1 & 0 & 5 & -1 \\ 2 & 2 & 1 & -2 \\ -2 & -2 & -4 & 3 \\ 0 & -5 & 0 & 1 \end{vmatrix} = 1$$

$$|A_3| = \begin{vmatrix} -1 & 0 & 0 & -1 \\ 2 & 1 & 2 & -2 \\ -2 & -1 & -2 & 3 \\ 0 & -2 & -5 & 1 \end{vmatrix} = -1$$

$$|A_4| = \begin{vmatrix} -1 & 0 & 5 & 0 \\ 2 & 1 & 1 & 2 \\ -2 & -1 & -4 & -2 \\ 0 & -2 & 0 & -5 \end{vmatrix} = -3$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-2}{1} = -2 \\ x_2 = \frac{|A_2|}{|A|} = \frac{1}{1} = 1 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-1}{1} = -1 \\ x_4 = \frac{|A_4|}{|A|} = \frac{-3}{1} = -3 \end{cases}$$

A.4

$$|A| = \begin{vmatrix} 17 & -16 & 4 \\ -10 & 10 & -3 \\ -4 & 3 & 0 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} 79 & -16 & 4 \\ -47 & 10 & -3 \\ -18 & 3 & 0 \end{vmatrix} = 3$$

$$|A_2| = \begin{vmatrix} 17 & 79 & 4 \\ -10 & -47 & -3 \\ -4 & -18 & 0 \end{vmatrix} = -2$$

$$|A_3| = \begin{vmatrix} 17 & -16 & 79 \\ -10 & 10 & -47 \\ -4 & 3 & -18 \end{vmatrix} = -1$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{3}{1} = 3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-2}{1} = -2 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-1}{1} = -1 \end{cases}$$

A.5

$$|A| = \begin{vmatrix} 4 & 7 & -9 \\ 0 & -3 & 2 \\ -3 & -4 & 6 \end{vmatrix} = -1$$

$$|A_1| = \begin{vmatrix} -11 & 7 & -9 \\ 8 & -3 & 2 \\ 5 & -4 & 6 \end{vmatrix} = -3$$

$$|A_2| = \begin{vmatrix} 4 & -11 & -9 \\ 0 & 8 & 2 \\ -3 & 5 & 6 \end{vmatrix} = 2$$

$$|A_3| = \begin{vmatrix} 4 & 7 & -11 \\ 0 & -3 & 8 \\ -3 & -4 & 5 \end{vmatrix} = -1$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-3}{-1} = 3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{2}{-1} = -2 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-1}{-1} = 1 \end{cases}$$

## A.6

$$|A| = \begin{vmatrix} 1 & -2 & 0 \\ 1 & 2 & -1 \\ -1 & -1 & 1 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} -8 & -2 & 0 \\ 2 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -2$$

$$|A_2| = \begin{vmatrix} 1 & -8 & 0 \\ 1 & 2 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 3$$

$$|A_3| = \begin{vmatrix} 1 & -2 & -8 \\ 1 & 2 & 2 \\ -1 & -1 & 1 \end{vmatrix} = 2$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-2}{1} = -2 \\ x_2 = \frac{|A_2|}{|A|} = \frac{3}{1} = 3 \\ x_3 = \frac{|A_3|}{|A|} = \frac{2}{1} = 2 \end{cases}$$

## A.7

$$|A| = \begin{vmatrix} 2 & -1 & -2 & -1 \\ 0 & -3 & 3 & 1 \\ -1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{vmatrix} = -1$$

$$|A_1| = \begin{vmatrix} -1 & -1 & -2 & -1 \\ 4 & -3 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{vmatrix} = 4$$

$$|A_2| = \begin{vmatrix} 2 & -1 & -2 & -1 \\ 0 & 4 & 3 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{vmatrix} = 3$$

$$|A_3| = \begin{vmatrix} 2 & -1 & -1 & -1 \\ 0 & -3 & 4 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 1 \end{vmatrix} = 1$$

$$|A_4| = \begin{vmatrix} 2 & -1 & -2 & -1 \\ 0 & -3 & 3 & 4 \\ -1 & 1 & 1 & 0 \\ -1 & 1 & 0 & -1 \end{vmatrix} = 2$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{4}{-1} = -4 \\ x_2 = \frac{|A_2|}{|A|} = \frac{3}{-1} = -3 \\ x_3 = \frac{|A_3|}{|A|} = \frac{1}{-1} = -1 \\ x_4 = \frac{|A_4|}{|A|} = \frac{2}{-1} = -2 \end{cases}$$

A.8

$$|A| = \begin{vmatrix} -7 & 5 & 3 \\ 10 & -10 & -3 \\ 9 & -6 & -4 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} 32 & 5 & 3 \\ -53 & -10 & -3 \\ -40 & -6 & -4 \end{vmatrix} = -2$$

$$|A_2| = \begin{vmatrix} -7 & 32 & 3 \\ 10 & -53 & -3 \\ 9 & -40 & -4 \end{vmatrix} = 3$$

$$|A_3| = \begin{vmatrix} -7 & 5 & 32 \\ 10 & -10 & -53 \\ 9 & -6 & -40 \end{vmatrix} = 1$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-2}{1} = -2 \\ x_2 = \frac{|A_2|}{|A|} = \frac{3}{1} = 3 \\ x_3 = \frac{|A_3|}{|A|} = \frac{1}{1} = 1 \end{cases}$$

A.9

$$|A| = \begin{vmatrix} 1 & 4 & -3 & 0 \\ 1 & -3 & 0 & 0 \\ -1 & 0 & -1 & 1 \\ -1 & 1 & 1 & 0 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} -29 & 4 & -3 & 0 \\ 8 & -3 & 0 & 0 \\ 2 & 0 & -1 & 1 \\ 3 & 1 & 1 & 0 \end{vmatrix} = -4$$

$$|A_2| = \begin{vmatrix} 1 & -29 & -3 & 0 \\ 1 & 8 & 0 & 0 \\ -1 & 2 & -1 & 1 \\ -1 & 3 & 1 & 0 \end{vmatrix} = -4$$

$$|A_3| = \begin{vmatrix} 1 & 4 & -29 & 0 \\ 1 & -3 & 8 & 0 \\ -1 & 0 & 2 & 1 \\ -1 & 1 & 3 & 0 \end{vmatrix} = 3$$

$$|A_4| = \begin{vmatrix} 1 & 4 & -3 & -29 \\ 1 & -3 & 0 & 8 \\ -1 & 0 & -1 & 2 \\ -1 & 1 & 1 & 3 \end{vmatrix} = 1$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-4}{1} = -4 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-4}{1} = -4 \\ x_3 = \frac{|A_3|}{|A|} = \frac{3}{1} = 3 \\ x_4 = \frac{|A_4|}{|A|} = \frac{1}{1} = 1 \end{cases}$$

A.10

$$|A| = \begin{vmatrix} 8 & 14 & -13 \\ -13 & -22 & 21 \\ 2 & 3 & -3 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} -33 & 14 & -13 \\ 54 & -22 & 21 \\ -8 & 3 & -3 \end{vmatrix} = -1$$

$$|A_2| = \begin{vmatrix} 8 & -33 & -13 \\ -13 & 54 & 21 \\ 2 & -8 & -3 \end{vmatrix} = 1$$

$$|A_3| = \begin{vmatrix} 8 & 14 & -33 \\ -13 & -22 & 54 \\ 2 & 3 & -8 \end{vmatrix} = 3$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-1}{1} = -1 \\ x_2 = \frac{|A_2|}{|A|} = \frac{1}{1} = 1 \\ x_3 = \frac{|A_3|}{|A|} = \frac{3}{1} = 3 \end{cases}$$

A.11

$$|A| = \begin{vmatrix} 3 & -4 & 2 \\ -3 & 4 & -1 \\ -2 & 3 & -2 \end{vmatrix} = -1$$

$$|A_1| = \begin{vmatrix} 23 & -4 & 2 \\ -20 & 4 & -1 \\ -18 & 3 & -2 \end{vmatrix} = -3$$

$$|A_2| = \begin{vmatrix} 3 & 23 & 2 \\ -3 & -20 & -1 \\ -2 & -18 & -2 \end{vmatrix} = 2$$

$$|A_3| = \begin{vmatrix} 3 & -4 & 23 \\ -3 & 4 & -20 \\ -2 & 3 & -18 \end{vmatrix} = -3$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-3}{-1} = 3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{2}{-1} = -2 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-3}{-1} = 3 \end{cases}$$

A.12

$$|A| = \begin{vmatrix} -3 & 3 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -2 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} 14 & 3 & 1 \\ -5 & -1 & 0 \\ -1 & 0 & -2 \end{vmatrix} = -3$$

$$|A_2| = \begin{vmatrix} -3 & 14 & 1 \\ 1 & -5 & 0 \\ 1 & -1 & -2 \end{vmatrix} = 2$$

$$|A_3| = \begin{vmatrix} -3 & 3 & 14 \\ 1 & -1 & -5 \\ 1 & 0 & -1 \end{vmatrix} = -1$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{-3}{1} = -3 \\ x_2 = \frac{|A_2|}{|A|} = \frac{2}{1} = 2 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-1}{1} = -1 \end{cases}$$

A.13

$$|A| = \begin{vmatrix} -2 & 1 & 0 \\ -1 & 2 & -5 \\ 3 & -2 & 2 \end{vmatrix} = -1$$

$$|A_1| = \begin{vmatrix} 11 & 1 & 0 \\ -5 & 2 & -5 \\ -12 & -2 & 2 \end{vmatrix} = 4$$

$$|A_2| = \begin{vmatrix} -2 & 11 & 0 \\ -1 & -5 & -5 \\ 3 & -12 & 2 \end{vmatrix} = -3$$

$$|A_3| = \begin{vmatrix} -2 & 1 & 11 \\ -1 & 2 & -5 \\ 3 & -2 & -12 \end{vmatrix} = -3$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{4}{-1} = -4 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-3}{-1} = 3 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-3}{-1} = 3 \end{cases}$$

A.14

$$|A| = \begin{vmatrix} 11 & -9 & -7 & 4 \\ -2 & 2 & 1 & -1 \\ 1 & 0 & 2 & -2 \\ -3 & 2 & 1 & 0 \end{vmatrix} = 1$$

$$|A_1| = \begin{vmatrix} 41 & -9 & -7 & 4 \\ -7 & 2 & 1 & -1 \\ 0 & 0 & 2 & -2 \\ -10 & 2 & 1 & 0 \end{vmatrix} = 2$$

$$|A_2| = \begin{vmatrix} 11 & 41 & -7 & 4 \\ -2 & -7 & 1 & -1 \\ 1 & 0 & 2 & -2 \\ -3 & -10 & 1 & 0 \end{vmatrix} = -1$$

$$|A_3| = \begin{vmatrix} 11 & -9 & 41 & 4 \\ -2 & 2 & -7 & -1 \\ 1 & 0 & 0 & -2 \\ -3 & 2 & -10 & 0 \end{vmatrix} = -2$$

$$|A_4| = \begin{vmatrix} 11 & -9 & -7 & 41 \\ -2 & 2 & 1 & -7 \\ 1 & 0 & 2 & 0 \\ -3 & 2 & 1 & -10 \end{vmatrix} = -1$$

$$\begin{cases} x_1 = \frac{|A_1|}{|A|} = \frac{2}{1} = 2 \\ x_2 = \frac{|A_2|}{|A|} = \frac{-1}{1} = -1 \\ x_3 = \frac{|A_3|}{|A|} = \frac{-2}{1} = -2 \\ x_4 = \frac{|A_4|}{|A|} = \frac{-1}{1} = -1 \end{cases}$$