代数学幾何学 (A/B) 計算演習 [問題] (2009/11/19)

問. 次の二つの複素ベクトル u, v の内積 (u, v) を求めなさい

Q.1

$$u = \begin{pmatrix} -3+i \\ 1-i \\ 2 \\ 3-3i \\ 3+i \\ 3i \\ -3+2i \end{pmatrix},\ v = \begin{pmatrix} i \\ 2-2i \\ 3-i \\ 3-i \\ 2+i \\ -1 \\ 0 \end{pmatrix}$$

Q.2

$$u = \begin{pmatrix} -3+3i \\ 2-3i \\ -3+2i \\ 2-i \\ 1 \end{pmatrix},\ v = \begin{pmatrix} 2 \\ -2+i \\ -1+i \\ -3-3i \\ -3-i \end{pmatrix}$$

Q.3

$$u = \begin{pmatrix} -2-2i \\ -3-2i \\ 2 \\ 2-i \\ 3+3i \end{pmatrix},\ v = \begin{pmatrix} -1-i \\ -1-3i \\ 2+2i \\ -3-i \\ -2-i \end{pmatrix}$$

Q.4

$$u = \begin{pmatrix} 1-3i \\ -3+2i \\ -2-i \\ -3-2i \\ -3+i \\ -1-2i \end{pmatrix},\ v = \begin{pmatrix} 1-3i \\ 1+i \\ -1+i \\ -3i \\ 1-2i \\ 2+3i \end{pmatrix}$$

Q.5

$$u = \begin{pmatrix} 1-i \\ 3+2i \\ -3i \\ -3+2i \\ 3-3i \\ -3i \end{pmatrix},\ v = \begin{pmatrix} -2-2i \\ -2-2i \\ 2i \\ -1 \\ 2-i \\ -2-3i \end{pmatrix}$$

Q.6

$$u = \begin{pmatrix} -3+i \\ -2 \\ 1-2i \\ -2-2i \\ -1+i \\ 1 \\ i \end{pmatrix},\ v = \begin{pmatrix} -1+2i \\ 3-i \\ 0 \\ 1+3i \\ -1-i \\ 2+3i \\ -2+3i \end{pmatrix}$$

代数学幾何学 (A/B) 計算演習 [解答] (2009/11/19)

A.1

$$
\begin{aligned}
\left(\begin{pmatrix} -3+i \\ 1-i \\ 2 \\ 3-3i \\ 3+i \\ 3i \\ -3+2i \end{pmatrix}, \begin{pmatrix} i \\ 2-2i \\ 3-i \\ 3-i \\ 2+i \\ -1 \\ 0 \end{pmatrix}\right) &= \quad (-3+i)\times\overline{(i)} + (1-i)\times\overline{(2-2i)} \\
&+ \quad (2)\times\overline{(3-i)} + (3-3i)\times\overline{(3-i)} \\
&+ \quad (3+i)\times\overline{(2+i)} + (3i)\times\overline{(-1)} \\
&+ \quad (-3+2i)\times\overline{(0)} \\
&= \quad (-3+i)\times(-i) + (1-i)\times(2+2i) \\
&+ \quad (2)\times(3+i) + (3-3i)\times(3+i) \\
&+ \quad (3+i)\times(2-i) + (3i)\times(-1) \\
&+ \quad (-3+2i)\times(0) \\
&= \quad (1+3i) + (4) + (6+2i) + (12-6i) \\
&+ \quad (7-i) + (-3i) + (0) \\
&= \quad 30-5i
\end{aligned}
$$

A.2

$$
\begin{aligned}
\left(\begin{pmatrix} -3+3i \\ 2-3i \\ -3+2i \\ 2-i \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2+i \\ -1+i \\ -3-3i \\ -3-i \end{pmatrix}\right) &= \quad (-3+3i)\times\overline{(2)} + (2-3i)\times\overline{(-2+i)} \\
&+ \quad (-3+2i)\times\overline{(-1+i)} + (2-i)\times\overline{(-3-3i)} \\
&+ \quad (1)\times\overline{(-3-i)} \\
&= \quad (-3+3i)\times(2) + (2-3i)\times(-2-i) \\
&+ \quad (-3+2i)\times(-1-i) + (2-i)\times(-3+3i) \\
&+ \quad (1)\times(-3+i) \\
&= \quad (-6+6i) + (-7+4i) + (5+i) + (-3+9i) \\
&+ \quad (-3+i) \\
&= \quad -14+21i
\end{aligned}
$$

A.3

$$
\begin{aligned}
(\begin{pmatrix} -2-2i \\ -3-2i \\ 2 \\ 2-i \\ 3+3i \end{pmatrix}, \begin{pmatrix} -1-i \\ -1-3i \\ 2+2i \\ -3-i \\ -2-i \end{pmatrix}) &= (-2-2i) \times \overline{(-1-i)} + (-3-2i) \times \overline{(-1-3i)} \\
&+ (2) \times \overline{(2+2i)} + (2-i) \times \overline{(-3-i)} \\
&+ (3+3i) \times \overline{(-2-i)} \\
&= (-2-2i) \times (-1+i) + (-3-2i) \times (-1+3i) \\
&+ (2) \times (2-2i) + (2-i) \times (-3+i) \\
&+ (3+3i) \times (-2+i) \\
&= (4) + (9-7i) + (4-4i) + (-5+5i) \\
&+ (-9-3i) \\
&= 3-9i
\end{aligned}
$$

A.4

$$
\begin{aligned}
(\begin{pmatrix} 1-3i \\ -3+2i \\ -2-i \\ -3-2i \\ -3+i \\ -1-2i \end{pmatrix}, \begin{pmatrix} 1-3i \\ 1+i \\ -1+i \\ -3i \\ 1-2i \\ 2+3i \end{pmatrix}) &= (1-3i) \times \overline{(1-3i)} + (-3+2i) \times \overline{(1+i)} \\
&+ (-2-i) \times \overline{(-1+i)} + (-3-2i) \times \overline{(-3i)} \\
&+ (-3+i) \times \overline{(1-2i)} + (-1-2i) \times \overline{(2+3i)} \\
&= (1-3i) \times (1+3i) + (-3+2i) \times (1-i) \\
&+ (-2-i) \times (-1-i) + (-3-2i) \times (3i) \\
&+ (-3+i) \times (1+2i) + (-1-2i) \times (2-3i) \\
&= (10) + (-1+5i) + (1+3i) + (6-9i) \\
&+ (-5-5i) + (-8-i) \\
&= 3-7i
\end{aligned}
$$

A.5

$$\left(\begin{pmatrix} 1-i \\ 3+2i \\ -3i \\ -3+2i \\ 3-3i \\ -3i \end{pmatrix}, \begin{pmatrix} -2-2i \\ -2-2i \\ 2i \\ -1 \\ 2-i \\ -2-3i \end{pmatrix}\right) \begin{aligned}[t] = \quad & (1-i)\times\overline{(-2-2i)} + (3+2i)\times\overline{(-2-2i)} \\ + \quad & (-3i)\times\overline{(2i)} + (-3+2i)\times\overline{(-1)} \\ + \quad & (3-3i)\times\overline{(2-i)} + (-3i)\times\overline{(-2-3i)} \\ = \quad & (1-i)\times(-2+2i) + (3+2i)\times(-2+2i) \\ + \quad & (-3i)\times(-2i) + (-3+2i)\times(-1) \\ + \quad & (3-3i)\times(2+i) + (-3i)\times(-2+3i) \\ = \quad & (4i) + (-10+2i) + (-6) + (3-2i) \\ + \quad & (9-3i) + (9+6i) \\ = \quad & 5+7i \end{aligned}$$

A.6

$$\left(\begin{pmatrix} -3+i \\ -2 \\ 1-2i \\ -2-2i \\ -1+i \\ 1 \\ i \end{pmatrix}, \begin{pmatrix} -1+2i \\ 3-i \\ 0 \\ 1+3i \\ -1-i \\ 2+3i \\ -2+3i \end{pmatrix}\right) \begin{aligned}[t] = \quad & (-3+i)\times\overline{(-1+2i)} + (-2)\times\overline{(3-i)} \\ + \quad & (1-2i)\times\overline{(0)} + (-2-2i)\times\overline{(1+3i)} \\ + \quad & (-1+i)\times\overline{(-1-i)} + (1)\times\overline{(2+3i)} \\ + \quad & (i)\times\overline{(-2+3i)} \\ = \quad & (-3+i)\times(-1-2i) + (-2)\times(3+i) \\ + \quad & (1-2i)\times(0) + (-2-2i)\times(1-3i) \\ + \quad & (-1+i)\times(-1+i) + (1)\times(2-3i) \\ + \quad & (i)\times(-2-3i) \\ = \quad & (5+5i) + (-6-2i) + (0) + (-8+4i) \\ + \quad & (-2i) + (2-3i) + (3-2i) \\ = \quad & -4 \end{aligned}$$