

代数学幾何学 (A/B) 計算演習 [問題] (2009/12/03)

問. 次のベクトルが独立かどうかを調べ、独立でない場合は、自明でない線型関係を示せ。

Q.1

$$\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Q.8

$$\begin{pmatrix} 1 \\ -1 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \\ -3 \\ -8 \end{pmatrix}, \begin{pmatrix} 5 \\ -11 \\ -2 \\ -13 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

Q.2

$$\begin{pmatrix} -2 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -7 \\ 2 \\ -4 \\ 6 \end{pmatrix}, \begin{pmatrix} 10 \\ -4 \\ 6 \\ -10 \end{pmatrix}, \begin{pmatrix} -5 \\ 2 \\ -3 \\ 5 \end{pmatrix}$$

Q.9

$$\begin{pmatrix} -11 \\ -7 \\ -1 \\ -7 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 16 \\ 8 \\ 0 \\ 8 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \\ 1 \\ 4 \end{pmatrix}$$

Q.3

$$\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -5 \\ 2 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

Q.10

$$\begin{pmatrix} -8 \\ -2 \\ -11 \\ 8 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \\ -3 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ -2 \\ 2 \end{pmatrix}$$

Q.4

$$\begin{pmatrix} 0 \\ -2 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ -4 \\ -7 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -3 \\ -5 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

Q.11

$$\begin{pmatrix} -2 \\ 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ -7 \\ -5 \\ -8 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -2 \\ -2 \end{pmatrix}$$

Q.5

$$\begin{pmatrix} 5 \\ 5 \\ -2 \\ 10 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \\ 2 \\ -5 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ -3 \\ 9 \end{pmatrix}$$

Q.12

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -2 \\ -4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ -3 \end{pmatrix}$$

Q.6

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ -2 \\ -3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

Q.13

$$\begin{pmatrix} 1 \\ 5 \\ -3 \\ -7 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ -1 \\ -6 \end{pmatrix}, \begin{pmatrix} 1 \\ 6 \\ -2 \\ -7 \end{pmatrix}$$

Q.7

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 6 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Q.14

$$\begin{pmatrix} 1 \\ 0 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix}$$

代数学幾何学 (A/B) 計算演習 [解答] (2009/12/03)

A.1

$$\text{従属: } \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

A.2

$$\text{従属: } \begin{pmatrix} 10 \\ -4 \\ 6 \\ -10 \end{pmatrix} + 2 \begin{pmatrix} -5 \\ 2 \\ -3 \\ 5 \end{pmatrix} = 0$$

A.3

独立

A.4

$$\text{従属: } \begin{pmatrix} 0 \\ -2 \\ 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ -3 \\ -5 \end{pmatrix} = 0$$

A.5

$$\text{従属: } \begin{pmatrix} 5 \\ 5 \\ -2 \\ 10 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix} - 2 \begin{pmatrix} -2 \\ -3 \\ 2 \\ -5 \end{pmatrix} - 2 \begin{pmatrix} 4 \\ 5 \\ -3 \\ 9 \end{pmatrix} = 0$$

A.6

独立

A.7

独立

A.8

独立

A.9

$$\text{従属: } 2 \begin{pmatrix} -11 \\ -7 \\ -1 \\ -7 \end{pmatrix} + 2 \begin{pmatrix} -2 \\ -1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 16 \\ 8 \\ 0 \\ 8 \end{pmatrix} + 2 \begin{pmatrix} 5 \\ 4 \\ 1 \\ 4 \end{pmatrix} = 0$$

A.10

$$\text{従属: } \begin{pmatrix} -8 \\ -2 \\ -11 \\ 8 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 1 \\ 4 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ -1 \\ 2 \end{pmatrix} - 2 \begin{pmatrix} -2 \\ 0 \\ -2 \\ 2 \end{pmatrix} = 0$$

A.11

$$\text{従属: } - \begin{pmatrix} -2 \\ 3 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ -2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ -7 \\ -5 \\ -8 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ -1 \\ -2 \\ -2 \end{pmatrix} = 0$$

A.12

$$\text{従属: } -2 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ -2 \\ -4 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ 1 \\ -2 \\ -3 \end{pmatrix} = 0$$

A.13

$$\text{従属: } \begin{pmatrix} 1 \\ 5 \\ -3 \\ -7 \end{pmatrix} - 2 \begin{pmatrix} 0 \\ -3 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ -1 \\ -6 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \\ -2 \\ -7 \end{pmatrix} = 0$$

A.14

$$\text{従属: } - \begin{pmatrix} 1 \\ 0 \\ -1 \\ -1 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix} = 0$$