

# 代数学幾何学 (A/B) 計算演習 [問題] (2009/12/10)

問. 次の二組の基底の変換行列を求めなさい。

Q.1

$$E = \left\langle \begin{pmatrix} -1 \\ -5 \\ -1 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 12 \\ 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 15 \\ 4 \\ 9 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} 9 \\ 31 \\ 7 \\ 19 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -12 \\ -41 \\ -9 \\ -25 \end{pmatrix}, \begin{pmatrix} 11 \\ 38 \\ 9 \\ 23 \end{pmatrix} \right\rangle$$

Q.5

$$E = \left\langle \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} -10 \\ 1 \\ -11 \\ -2 \end{pmatrix}, \begin{pmatrix} 6 \\ -2 \\ 16 \\ -3 \end{pmatrix}, \begin{pmatrix} -5 \\ 5 \\ -37 \\ 13 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 6 \\ -2 \end{pmatrix} \right\rangle$$

Q.2

$$E = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} -5 \\ -6 \\ -45 \\ -5 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 6 \\ 43 \\ 8 \end{pmatrix}, \begin{pmatrix} -2 \\ -4 \\ -27 \\ -5 \end{pmatrix} \right\rangle$$

Q.6

$$E = \left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -2 \\ -1 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} -3 \\ -3 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} -5 \\ -5 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ -4 \\ 0 \end{pmatrix} \right\rangle$$

Q.3

$$E = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -6 \\ -5 \\ -5 \\ -6 \end{pmatrix} \right\rangle$$

Q.7

$$E = \left\langle \begin{pmatrix} 0 \\ 0 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -2 \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} 7 \\ -1 \\ -4 \\ -4 \end{pmatrix}, \begin{pmatrix} -5 \\ 5 \\ -7 \\ -15 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ -5 \\ -10 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ -2 \\ -2 \end{pmatrix} \right\rangle$$

Q.4

$$E = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ -2 \\ 7 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 1 \\ -6 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \\ 4 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} -8 \\ 21 \\ 9 \\ -39 \end{pmatrix}, \begin{pmatrix} 6 \\ -18 \\ -8 \\ 33 \end{pmatrix}, \begin{pmatrix} 2 \\ -6 \\ -3 \\ 11 \end{pmatrix}, \begin{pmatrix} -5 \\ 14 \\ 6 \\ -26 \end{pmatrix} \right\rangle$$

Q.8

$$E = \left\langle \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -5 \\ -9 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \\ 2 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} -16 \\ -31 \\ 5 \\ -5 \end{pmatrix}, \begin{pmatrix} 20 \\ 38 \\ -6 \\ 7 \end{pmatrix}, \begin{pmatrix} -9 \\ -18 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} -4 \\ -8 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

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A.1

$$\begin{pmatrix} -3 & 1 & 4 & -3 \\ -2 & 1 & 3 & -1 \\ 0 & 1 & -1 & 0 \\ 2 & -1 & -2 & 2 \end{pmatrix}$$

A.5

$$\begin{pmatrix} 2 & -3 & 7 & -1 \\ -5 & 4 & -6 & 1 \\ -2 & -3 & 13 & -2 \\ -3 & 5 & -12 & 2 \end{pmatrix}$$

A.2

$$\begin{pmatrix} -1 & 3 & 3 & -2 \\ 10 & -2 & -11 & 7 \\ -5 & -2 & 3 & -2 \\ 5 & 3 & -4 & 2 \end{pmatrix}$$

A.6

$$\begin{pmatrix} -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & -2 & 1 \end{pmatrix}$$

A.3

$$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 2 \\ 0 & 1 & 1 & -2 \end{pmatrix}$$

A.7

$$\begin{pmatrix} 2 & -2 & -1 & 0 \\ 1 & 3 & 2 & 1 \\ 2 & -2 & -1 & 1 \\ -2 & -1 & -1 & -1 \end{pmatrix}$$

A.4

$$\begin{pmatrix} -4 & 2 & 1 & -2 \\ -3 & 3 & 1 & -2 \\ 3 & -2 & -1 & 2 \\ -2 & 1 & 0 & -1 \end{pmatrix}$$

A.8

$$\begin{pmatrix} 5 & -5 & 4 & 2 \\ 3 & -3 & 2 & 1 \\ -3 & 4 & -2 & -1 \\ 1 & -1 & 1 & 1 \end{pmatrix}$$