

代数学幾何学 (A/B) 計算演習 [問題] (2009/12/10)

問. 線型空間 W の、次の二組の基底の変換行列を求めなさい。

Q.1

$$\begin{aligned}
 W &= \{v \in V^5 \mid 3v_1 + v_2 - v_3 + 2v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} 2 \\ 0 \\ -1 \\ 1 \\ -9 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 3 \\ -8 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ -2 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} -4 \\ 2 \\ 3 \\ 2 \\ 9 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ -1 \\ 5 \\ -28 \end{pmatrix}, \begin{pmatrix} 5 \\ -1 \\ -2 \\ 2 \\ -20 \end{pmatrix}, \begin{pmatrix} -5 \\ 2 \\ 5 \\ 4 \\ 10 \end{pmatrix} \right\rangle
 \end{aligned}$$

Q.2

$$\begin{aligned}
 W &= \{v \in V^5 \mid -3v_1 + 3v_2 + v_3 - v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} 31 \\ -10 \\ 3 \\ -19 \\ 101 \end{pmatrix}, \begin{pmatrix} -54 \\ 17 \\ -5 \\ 33 \\ -175 \end{pmatrix}, \begin{pmatrix} -15 \\ 3 \\ -1 \\ 9 \\ -44 \end{pmatrix}, \begin{pmatrix} 10 \\ -3 \\ 1 \\ -6 \\ 32 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} 56 \\ -16 \\ 5 \\ -34 \\ 177 \end{pmatrix}, \begin{pmatrix} -173 \\ 55 \\ -16 \\ 106 \\ -562 \end{pmatrix}, \begin{pmatrix} 98 \\ -31 \\ 9 \\ -60 \\ 318 \end{pmatrix}, \begin{pmatrix} -138 \\ 46 \\ -13 \\ 85 \\ -454 \end{pmatrix} \right\rangle
 \end{aligned}$$

Q.3

$$\begin{aligned}
 W &= \{v \in V^5 \mid 3v_1 - v_2 + 2v_3 + v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ -5 \end{pmatrix}, \begin{pmatrix} -3 \\ -4 \\ 2 \\ 3 \\ -2 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \\ 2 \\ -8 \end{pmatrix}, \begin{pmatrix} 7 \\ 10 \\ -6 \\ -8 \\ 9 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 0 \\ 1 \\ -9 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 1 \\ 3 \\ -19 \end{pmatrix} \right\rangle
 \end{aligned}$$

Q.4

$$\begin{aligned}
 W &= \{v \in V^5 \mid -v_1 + v_2 + 2v_3 + 3v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} -3 \\ 1 \\ -4 \\ -6 \\ 22 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \\ -7 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \\ 3 \\ -9 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} -5 \\ 1 \\ -4 \\ -9 \\ 29 \end{pmatrix}, \begin{pmatrix} 7 \\ -1 \\ 6 \\ 12 \\ -40 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \\ 2 \\ -9 \end{pmatrix}, \begin{pmatrix} -5 \\ 1 \\ -5 \\ -9 \\ 31 \end{pmatrix} \right\rangle
 \end{aligned}$$

Q.5

$$\begin{aligned}
 W &= \{v \in V^5 \mid v_1 - v_2 + 3v_3 + v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} 1 \\ 2 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \\ 1 \\ -4 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \\ -5 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} -4 \\ -36 \\ 1 \\ 23 \\ -58 \end{pmatrix}, \begin{pmatrix} -3 \\ -14 \\ -3 \\ 8 \\ -10 \end{pmatrix}, \begin{pmatrix} -3 \\ -20 \\ -2 \\ 12 \\ -23 \end{pmatrix}, \begin{pmatrix} -3 \\ -27 \\ 0 \\ 17 \\ -41 \end{pmatrix} \right\rangle
 \end{aligned}$$

Q.6

$$\begin{aligned}
 W &= \{v \in V^5 \mid -2v_1 + v_2 - v_3 + 2v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ 0 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} -16 \\ -12 \\ 3 \\ -19 \\ 21 \end{pmatrix}, \begin{pmatrix} -5 \\ -3 \\ 2 \\ -6 \\ 7 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ -1 \\ 5 \\ -6 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ -1 \\ 4 \\ -5 \end{pmatrix} \right\rangle
 \end{aligned}$$

Q.7

$$\begin{aligned}
 W &= \{v \in V^5 \mid 3v_1 - 3v_2 - v_3 - 3v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} 1 \\ 2 \\ 4 \\ 2 \\ 13 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \\ 2 \\ 10 \end{pmatrix}, \begin{pmatrix} 7 \\ 11 \\ 7 \\ 9 \\ 46 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} 45 \\ 70 \\ 42 \\ 56 \\ 285 \end{pmatrix}, \begin{pmatrix} -7 \\ -11 \\ -7 \\ -9 \\ -46 \end{pmatrix}, \begin{pmatrix} -5 \\ -9 \\ -12 \\ -9 \\ -51 \end{pmatrix}, \begin{pmatrix} 27 \\ 43 \\ 31 \\ 36 \\ 187 \end{pmatrix} \right\rangle
 \end{aligned}$$

Q.8

$$\begin{aligned}
 W &= \{v \in V^5 \mid 2v_1 - v_2 - 3v_3 + 3v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} -1 \\ -3 \\ -4 \\ 1 \\ -16 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ -3 \\ 1 \\ -12 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 6 \\ 0 \\ 20 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} 17 \\ 63 \\ 98 \\ -7 \\ 344 \end{pmatrix}, \begin{pmatrix} 31 \\ 114 \\ 177 \\ -13 \\ 622 \end{pmatrix}, \begin{pmatrix} -25 \\ -91 \\ -141 \\ 11 \\ -497 \end{pmatrix}, \begin{pmatrix} 26 \\ 96 \\ 149 \\ -11 \\ 524 \end{pmatrix} \right\rangle
 \end{aligned}$$

Q.9

$$\begin{aligned}
 W &= \{v \in V^5 \mid -v_1 + 3v_2 - 3v_3 - 2v_4 + v_5 = 0\} \\
 E &= \left\langle \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \\ -3 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \right\rangle \\
 F &= \left\langle \begin{pmatrix} -8 \\ -3 \\ -4 \\ 3 \\ -5 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 3 \\ -2 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -11 \\ -2 \\ -6 \\ 4 \\ -15 \end{pmatrix} \right\rangle
 \end{aligned}$$

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A.1

$$\begin{pmatrix} -1 & 2 & 2 & -2 \\ 1 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix}$$

A.2

$$\begin{pmatrix} 1 & -1 & 0 & -1 \\ 0 & 3 & -2 & 3 \\ -1 & 0 & 0 & -1 \\ 1 & 2 & -1 & 4 \end{pmatrix}$$

A.3

$$\begin{pmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 2 & 5 \\ 1 & -1 & 1 & 2 \\ 1 & -3 & 1 & 3 \end{pmatrix}$$

A.4

$$\begin{pmatrix} 1 & -1 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 2 & 1 & -1 \\ 2 & 0 & 1 & 0 \end{pmatrix}$$

A.5

$$\begin{pmatrix} -8 & -4 & -5 & -6 \\ -1 & -1 & -1 & -1 \\ 4 & 1 & 2 & 3 \\ 10 & 2 & 4 & 7 \end{pmatrix}$$

A.6

$$\begin{pmatrix} 3 & 1 & -1 & -1 \\ 4 & 1 & -1 & -1 \\ 3 & 2 & -1 & -1 \\ 5 & 1 & -1 & 0 \end{pmatrix}$$

A.7

$$\begin{pmatrix} 1 & 0 & -1 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 6 & -1 & -1 & 4 \end{pmatrix}$$

A.8

$$\begin{pmatrix} -1 & -2 & 2 & -2 \\ -14 & -25 & 20 & -21 \\ -8 & -14 & 11 & -12 \\ 10 & 18 & -14 & 15 \end{pmatrix}$$

A.9

$$\begin{pmatrix} 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & -2 \\ -2 & 1 & 0 & -3 \end{pmatrix}$$