

代数学幾何学 (A/B) 計算演習 [問題] (2009/12/17)

問. 次の行列 A によって定まる線型変換 T_A の基底 E, F に関する行列を求めなさい。

Q.1

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -6 & -3 & 1 & 2 \\ 2 & 1 & -1 & -1 \\ -1 & -2 & -1 & -2 \end{pmatrix}$$

$$E = \left\langle \begin{pmatrix} 2 \\ 2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -6 \\ -6 \\ 4 \\ -5 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

Q.2

$$A = \begin{pmatrix} -1 & 0 & 0 & -1 \\ -2 & -1 & 0 & -2 \\ 2 & 1 & 0 & 3 \\ 4 & 2 & 1 & 6 \end{pmatrix}$$

$$E = \left\langle \begin{pmatrix} 2 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 2 \\ -1 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} -1 \\ 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix} \right\rangle$$

Q.3

$$A = \begin{pmatrix} -1 & -1 & -1 & 0 \\ -2 & -1 & -2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$$E = \left\langle \begin{pmatrix} 1 \\ -1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 0 \\ -2 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -2 \\ -1 \end{pmatrix} \right\rangle$$

Q.4

$$A = \begin{pmatrix} 6 & -5 & -3 & -1 \\ -1 & 1 & 1 & 0 \\ 4 & -3 & -2 & 0 \\ -8 & 6 & 5 & 0 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} -1 \\ -1 \\ -2 \\ -6 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -1 \\ -6 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix} \right\rangle$$

Q.5

$$A = \begin{pmatrix} 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 2 & -1 & 0 & 1 \\ 5 & 0 & -1 & 1 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} -3 \\ -3 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 2 \\ 1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} -2 \\ -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 13 \\ 3 \\ 6 \\ 4 \end{pmatrix}, \begin{pmatrix} -6 \\ -1 \\ -3 \\ -2 \end{pmatrix} \right\rangle$$

Q.6

$$A = \begin{pmatrix} 3 & 2 & 1 & 1 \\ 4 & 3 & 2 & 1 \\ 0 & 2 & 1 & -1 \\ 6 & 3 & 2 & 2 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ -2 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 0 \\ -1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

Q.7

$$A = \begin{pmatrix} 0 & 1 & -3 & 0 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 2 & 0 \\ 1 & -1 & 6 & 0 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 3 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\rangle$$

Q.8

$$A = \begin{pmatrix} 2 & 1 & -1 & 1 \\ -2 & 0 & 1 & 1 \\ -8 & -1 & 4 & 3 \\ 9 & 2 & -4 & -3 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} 0 \\ 1 \\ -6 \\ -8 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -4 \\ -5 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \\ 0 \end{pmatrix} \right\rangle$$

Q.9

$$A = \begin{pmatrix} -1 & 0 & -1 & 1 \\ 2 & 1 & 2 & -2 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} -2 \\ -2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 9 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 5 \\ -3 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -4 \\ 1 \\ 2 \\ 0 \end{pmatrix} \right\rangle$$

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A.1

$$\begin{pmatrix} -18 & -12 & 67 & -14 \\ -1 & -1 & -6 & 3 \\ -11 & -8 & 24 & -2 \\ 16 & 12 & -35 & 3 \end{pmatrix}$$

A.2

$$\begin{pmatrix} 1 & 3 & 3 & 7 \\ -1 & 5 & 6 & 11 \\ 0 & -5 & -6 & -11 \\ 4 & -2 & -5 & -3 \end{pmatrix}$$

A.3

$$\begin{pmatrix} -7 & 13 & 5 & -5 \\ 0 & 3 & 1 & 2 \\ -2 & 6 & 2 & -1 \\ 4 & -11 & -4 & 1 \end{pmatrix}$$

A.4

$$\begin{pmatrix} -2 & 16 & 0 & 11 \\ 1 & -30 & -5 & -17 \\ 3 & 2 & 6 & -3 \\ 0 & 71 & 16 & 37 \end{pmatrix}$$

A.5

$$\begin{pmatrix} -45 & -17 & 28 & 62 \\ -27 & -14 & 17 & 37 \\ 1 & 3 & -1 & -2 \\ -9 & -1 & 5 & 11 \end{pmatrix}$$

A.6

$$\begin{pmatrix} 85 & 19 & -140 & 24 \\ 46 & 9 & -75 & 13 \\ 29 & 5 & -47 & 8 \\ 14 & 3 & -23 & 4 \end{pmatrix}$$

A.7

$$\begin{pmatrix} 1 & 1 & -1 & 3 \\ 3 & 0 & -1 & 0 \\ -1 & -2 & 2 & -6 \\ 2 & -2 & 2 & -7 \end{pmatrix}$$

A.8

$$\begin{pmatrix} -181 & -152 & 133 & -61 \\ 231 & 205 & -175 & 88 \\ 164 & 145 & -124 & 62 \\ 83 & 72 & -62 & 30 \end{pmatrix}$$

A.9

$$\begin{pmatrix} -30 & -21 & -46 & 7 \\ 8 & 7 & 19 & 8 \\ 9 & 7 & 16 & 2 \\ 3 & 3 & 8 & 5 \end{pmatrix}$$