代数学幾何学 (A/B) 計算演習 [問題] (2009/12/17)

問. 次の独立なベクトルから、シュミットの直交化を利用して、正規直交系を求めなさい

Q.1

$$< \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -5 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix} >$$

Q.2

$$< \begin{pmatrix} -1 \\ 0 \\ -2 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ -1 \end{pmatrix} >$$

Q.3

$$< \begin{pmatrix} 1 \\ -4 \\ 6 \\ 8 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -5 \\ 6 \\ 9 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 5 \\ 6 \end{pmatrix} >$$

Q.4

$$< \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ 0 \end{pmatrix} >$$

Q.5

$$< \begin{pmatrix} -2 \\ 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ -4 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} -4 \\ 1 \\ 3 \\ 4 \end{pmatrix} >$$

Q.6

$$< \begin{pmatrix} 5 \\ -2 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} >$$

代数学幾何学 (A/B) 計算演習 [解答] (2009/12/17)

A.1

$$< \begin{pmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{15}}{15} \\ -\frac{\sqrt{15}}{15} \\ \frac{2\sqrt{15}}{15} \\ -\frac{\sqrt{15}}{5} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{7}}{7} \\ -\frac{2\sqrt{7}}{7} \\ \frac{\sqrt{7}}{7} \\ \frac{\sqrt{7}}{7} \end{pmatrix}, \begin{pmatrix} \frac{4\sqrt{35}}{35} \\ \frac{\sqrt{35}}{35} \\ \frac{3\sqrt{35}}{35} \\ \frac{3\sqrt{35}}{35} \end{pmatrix} >$$

A.2

$$< \begin{pmatrix} -\frac{\sqrt{14}}{14} \\ 0 \\ -\frac{\sqrt{14}}{7} \\ -\frac{3\sqrt{14}}{14} \end{pmatrix}, \begin{pmatrix} -\frac{5\sqrt{91}}{91} \\ \frac{\sqrt{91}}{13} \\ \frac{4\sqrt{91}}{91} \\ -\frac{\sqrt{91}}{91} \end{pmatrix}, \begin{pmatrix} -\frac{6\sqrt{65}}{65} \\ -\frac{2\sqrt{65}}{65} \\ -\frac{3\sqrt{65}}{65} \\ \frac{4\sqrt{65}}{65} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{10}}{10} \\ \frac{\sqrt{10}}{5} \\ -\frac{\sqrt{10}}{5} \\ \frac{\sqrt{10}}{10} \end{pmatrix} >$$

A.3

$$< \begin{pmatrix} \frac{\sqrt{13}}{39} \\ -\frac{4\sqrt{13}}{39} \\ \frac{2\sqrt{13}}{13} \\ \frac{8\sqrt{13}}{39} \end{pmatrix}, \begin{pmatrix} -\frac{10\sqrt{221}}{663} \\ -\frac{38\sqrt{221}}{663} \\ -\frac{7\sqrt{221}}{221} \\ -\frac{2\sqrt{221}}{663} \end{pmatrix}, \begin{pmatrix} \frac{2\sqrt{34}}{51} \\ \frac{5\sqrt{34}}{102} \\ -\frac{2\sqrt{34}}{17} \\ \frac{11\sqrt{34}}{102} \end{pmatrix}, \begin{pmatrix} \frac{2\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{6} \\ 0 \\ -\frac{\sqrt{2}}{6} \end{pmatrix} >$$

A.4

$$< \begin{pmatrix} \frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{6} \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{2\sqrt{102}}{51} \\ \frac{7\sqrt{102}}{102} \\ \frac{\sqrt{102}}{102} \\ \frac{\sqrt{102}}{17} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{102}}{102} \\ \frac{\sqrt{102}}{17} \\ -\frac{2\sqrt{102}}{51} \\ -\frac{7\sqrt{102}}{102} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6} \end{pmatrix} >$$

A.5

$$< \begin{pmatrix} -\frac{\sqrt{10}}{5} \\ \frac{\sqrt{10}}{10} \\ \frac{\sqrt{10}}{5} \\ \frac{\sqrt{10}}{10} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{10}}{15} \\ \frac{\sqrt{10}}{30} \\ \frac{\sqrt{10}}{15} \\ -\frac{3\sqrt{10}}{10} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{5}}{3} \\ \frac{2\sqrt{5}}{15} \\ \frac{4\sqrt{5}}{15} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{2\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \\ 0 \end{pmatrix} >$$

A.6

$$< \begin{pmatrix} \frac{5\sqrt{34}}{34} \\ -\frac{\sqrt{34}}{17} \\ \frac{\sqrt{34}}{17} \\ -\frac{\sqrt{34}}{34} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{646}}{646} \\ \frac{10\sqrt{646}}{323} \\ \frac{7\sqrt{646}}{323} \\ -\frac{7\sqrt{646}}{646} \end{pmatrix}, \begin{pmatrix} \frac{4\sqrt{133}}{133} \\ \frac{4\sqrt{133}}{133} \\ -\frac{\sqrt{133}}{133} \\ \frac{10\sqrt{133}}{133} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{7}}{7} \\ -\frac{\sqrt{7}}{7} \\ \frac{2\sqrt{7}}{7} \\ \frac{\sqrt{7}}{7} \end{pmatrix} >$$