

代数学幾何学 (A/B) 計算演習 [問題] (2009/12/17)

問. 次の独立な複素ベクトルから、シュミットの直交化を利用して、正規直交系を求めなさい

Q.1

$$\left\langle \begin{pmatrix} i \\ 2 \\ -i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2+3i \\ 2+5i \\ 3-i \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2i \\ -1 \\ -i \end{pmatrix} \right\rangle$$

Q.2

$$\left\langle \begin{pmatrix} 3i \\ -2 \\ 1+i \\ 1+2i \end{pmatrix}, \begin{pmatrix} 2 \\ 2i \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} i \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -i \\ 1 \\ 0 \\ -i \end{pmatrix} \right\rangle$$

Q.3

$$\left\langle \begin{pmatrix} i \\ 1-i \\ 0 \\ 1+i \end{pmatrix}, \begin{pmatrix} -i \\ 1+2i \\ i \\ -1+i \end{pmatrix}, \begin{pmatrix} 0 \\ -i \\ -i \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ i \\ 0 \\ -1 \end{pmatrix} \right\rangle$$

Q.4

$$\left\langle \begin{pmatrix} -1+i \\ 2+i \\ 1+2i \\ 2+i \end{pmatrix}, \begin{pmatrix} -i \\ -1-i \\ -1-i \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1-i \\ 1 \\ -i \end{pmatrix}, \begin{pmatrix} -1-i \\ -2+i \\ -2+i \\ -1+i \end{pmatrix} \right\rangle$$

Q.5

$$\left\langle \begin{pmatrix} 0 \\ -i \\ 0 \\ -i \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1-i \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} i \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

Q.6

$$\left\langle \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}, \begin{pmatrix} i \\ -1+2i \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2+i \\ 1 \\ i \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ i \end{pmatrix} \right\rangle$$

代数学幾何学 (A/B) 計算演習 [解答] (2009/12/17)

A.1

$$\left\langle \begin{pmatrix} \frac{\sqrt{6}}{6}i \\ \frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6}i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{138}}{46} + \frac{5\sqrt{138}}{69}i \\ -\frac{2\sqrt{138}}{69} \\ \frac{\sqrt{138}}{46} + \frac{\sqrt{138}}{69}i \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{23}}{23} \\ \frac{2\sqrt{23}}{23} + \frac{\sqrt{23}}{23}i \\ -\frac{\sqrt{23}}{23} + \frac{4\sqrt{23}}{23}i \\ 0 \end{pmatrix} \right\rangle$$

A.2

$$\left\langle \begin{pmatrix} \frac{3\sqrt{5}}{10}i \\ -\frac{\sqrt{5}}{5} \\ \frac{\sqrt{5}}{10} + \frac{\sqrt{5}}{10}i \\ \frac{\sqrt{5}}{10} + \frac{\sqrt{5}}{5}i \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{130}}{52} - \frac{9\sqrt{130}}{260}i \\ \frac{3\sqrt{130}}{130} + \frac{\sqrt{130}}{26}i \\ \frac{\sqrt{130}}{130} + \frac{3\sqrt{130}}{65}i \\ \frac{7\sqrt{130}}{260} + \frac{9\sqrt{130}}{260}i \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{13} + \frac{3\sqrt{2}}{26}i \\ \frac{2\sqrt{2}}{13} + \frac{3\sqrt{2}}{13}i \\ \frac{5\sqrt{2}}{13} + \frac{\sqrt{2}}{13}i \\ -\frac{\sqrt{2}}{2} \end{pmatrix}, \begin{pmatrix} \frac{2\sqrt{13}}{13} \\ \frac{\sqrt{13}}{13} - \frac{2\sqrt{13}}{13}i \\ \frac{2\sqrt{13}}{13}i \\ 0 \end{pmatrix} \right\rangle$$

A.3

$$\left\langle \begin{pmatrix} \frac{\sqrt{5}}{5}i \\ \frac{\sqrt{5}}{5} - \frac{\sqrt{5}}{5}i \\ 0 \\ \frac{\sqrt{5}}{5} + \frac{\sqrt{5}}{5}i \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{5}}{4} - \frac{3\sqrt{5}}{20}i \\ \frac{\sqrt{5}}{10} + \frac{3\sqrt{5}}{20}i \\ \frac{\sqrt{5}}{4}i \\ \frac{\sqrt{5}}{10} + \frac{\sqrt{5}}{10}i \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{3}}{12} - \frac{\sqrt{3}}{4}i \\ \frac{\sqrt{3}}{12}i \\ -\frac{\sqrt{3}}{6} - \frac{5\sqrt{3}}{12}i \\ \frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{6}i \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{6}i \\ \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{6}i \\ -\frac{\sqrt{3}}{6}i \\ -\frac{\sqrt{3}}{3}i \end{pmatrix} \right\rangle$$

A.4

$$\left\langle \begin{pmatrix} -\frac{\sqrt{17}}{17} + \frac{\sqrt{17}}{17}i \\ \frac{2\sqrt{17}}{17} + \frac{\sqrt{17}}{17}i \\ \frac{\sqrt{17}}{17} + \frac{2\sqrt{17}}{17}i \\ \frac{2\sqrt{17}}{17} + \frac{\sqrt{17}}{17}i \end{pmatrix}, \begin{pmatrix} -\frac{7}{17} - \frac{6}{17}i \\ \frac{7}{17} - \frac{12}{17}i \\ -\frac{4}{17} - \frac{17}{17}i \\ \frac{3}{17} + \frac{5}{17}i \end{pmatrix}, \begin{pmatrix} -\frac{2}{17} - \frac{9}{17}i \\ \frac{7}{17} - \frac{17}{17}i \\ \frac{11}{17} + \frac{7}{17}i \\ -\frac{4}{17} - \frac{17}{17}i \end{pmatrix}, \begin{pmatrix} -\frac{2\sqrt{17}}{17} - \frac{\sqrt{17}}{17}i \\ \frac{\sqrt{17}}{17} + \frac{\sqrt{17}}{17}i \\ -\frac{\sqrt{17}}{17} \\ -\frac{3\sqrt{17}}{17}i \end{pmatrix} \right\rangle$$

A.5

$$\left\langle \begin{pmatrix} 0 \\ -\frac{\sqrt{2}}{2}i \\ 0 \\ -\frac{\sqrt{2}}{2}i \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{2}i \\ 0 \\ -\frac{\sqrt{2}}{2}i \\ 0 \end{pmatrix} \right\rangle$$

A.6

$$\left\langle \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2}i \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{3}i \\ -\frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{3}i \\ -\frac{\sqrt{2}}{3} - \frac{\sqrt{2}}{6}i \\ -\frac{\sqrt{2}}{3} \end{pmatrix}, \begin{pmatrix} \frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{3}i \\ -\frac{\sqrt{2}}{3}i \\ \frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} + \frac{\sqrt{2}}{6}i \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ 0 \\ \frac{\sqrt{2}}{2}i \end{pmatrix} \right\rangle$$