

代数学幾何学 (A/B) 計算演習 [問題] (2010/01/14)

問. 線型空間 (V^n) の、次の二組の基底 E, F に対して、 E から F への基底の変換行列を求めなさい。

Q.1

$$E = \left\langle \begin{pmatrix} 1 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ -3 \\ -3 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 1 \\ 0 \\ -4 \\ -3 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ -9 \\ -7 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ -16 \\ -9 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \\ -12 \\ -9 \end{pmatrix} \right\rangle$$

Q.2

$$E = \left\langle \begin{pmatrix} -1 \\ 4 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ -2 \\ -1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} -9 \\ 4 \\ -5 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ 5 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -9 \\ -1 \\ -6 \end{pmatrix}, \begin{pmatrix} -7 \\ -8 \\ -6 \\ -8 \end{pmatrix} \right\rangle$$

Q.3

$$E = \left\langle \begin{pmatrix} 7 \\ 4 \\ -8 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -3 \\ -1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 10 \\ 9 \\ -15 \\ -5 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ -4 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

Q.4

$$E = \left\langle \begin{pmatrix} 3 \\ 1 \\ 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 5 \\ 4 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 6 \\ 8 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} -4 \\ -1 \\ -5 \\ -7 \end{pmatrix} \right\rangle$$

Q.5

$$E = \left\langle \begin{pmatrix} 4 \\ -3 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ -3 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 41 \\ -25 \\ -1 \\ -8 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -19 \\ 11 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} -58 \\ 36 \\ 2 \\ 11 \end{pmatrix} \right\rangle$$

Q.6

$$E = \left\langle \begin{pmatrix} -17 \\ -11 \\ -29 \\ -7 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \\ -6 \\ -1 \end{pmatrix}, \begin{pmatrix} -15 \\ -10 \\ -26 \\ -6 \end{pmatrix}, \begin{pmatrix} 7 \\ 6 \\ 14 \\ 3 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} -17 \\ -13 \\ -32 \\ -7 \end{pmatrix}, \begin{pmatrix} 24 \\ 19 \\ 46 \\ 10 \end{pmatrix}, \begin{pmatrix} 19 \\ 18 \\ 41 \\ 8 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \\ -6 \\ -1 \end{pmatrix} \right\rangle$$

Q.7

$$E = \left\langle \begin{pmatrix} -4 \\ 10 \\ 8 \\ -7 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \\ 4 \\ -4 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \\ -3 \\ 2 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} -10 \\ 19 \\ 13 \\ -9 \end{pmatrix}, \begin{pmatrix} 30 \\ -58 \\ -39 \\ 31 \end{pmatrix}, \begin{pmatrix} -19 \\ 37 \\ 25 \\ -20 \end{pmatrix}, \begin{pmatrix} -9 \\ 17 \\ 11 \\ -9 \end{pmatrix} \right\rangle$$

Q.8

$$E = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 1 \\ -6 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 5 \\ -3 \\ 11 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 1 \\ 3 \\ -1 \\ 6 \end{pmatrix}, \begin{pmatrix} 6 \\ 5 \\ -1 \\ 9 \end{pmatrix}, \begin{pmatrix} 7 \\ 4 \\ -1 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 1 \\ -6 \end{pmatrix} \right\rangle$$

Q.9

$$E = \left\langle \begin{pmatrix} -1 \\ 4 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 1 \\ -7 \\ -2 \\ 4 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -6 \\ 13 \\ 3 \\ -7 \end{pmatrix}, \begin{pmatrix} 2 \\ -7 \\ -2 \\ 4 \end{pmatrix} \right\rangle$$

Q.10

$$E = \left\langle \begin{pmatrix} -2 \\ 10 \\ -2 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 0 \\ -4 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ -36 \\ 11 \\ -19 \end{pmatrix}, \begin{pmatrix} -2 \\ -65 \\ 20 \\ -34 \end{pmatrix}, \begin{pmatrix} 1 \\ 55 \\ -17 \\ 29 \end{pmatrix} \right\rangle$$

Q.11

$$E = \left\langle \begin{pmatrix} 3 \\ -1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \\ -2 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 16 \\ -7 \\ -14 \\ 22 \end{pmatrix}, \begin{pmatrix} -21 \\ 8 \\ 21 \\ -30 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 9 \\ -3 \\ -8 \\ 11 \end{pmatrix} \right\rangle$$

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A.1

$$\begin{pmatrix} 1 & 0 & -3 & -1 \\ 1 & 1 & 2 & 1 \\ -1 & -2 & 0 & -2 \\ 1 & 3 & 4 & 4 \end{pmatrix}$$

A.2

$$\begin{pmatrix} -1 & 0 & -1 & -1 \\ -3 & -2 & 2 & 2 \\ -2 & -1 & 1 & 0 \\ 0 & -1 & 2 & 4 \end{pmatrix}$$

A.3

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & -2 & -1 & 1 \\ -1 & -1 & 1 & 0 \\ 0 & 3 & 1 & -1 \end{pmatrix}$$

A.4

$$\begin{pmatrix} 1 & 0 & 1 & -1 \\ 1 & -1 & 1 & 0 \\ 3 & -3 & 2 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

A.5

$$\begin{pmatrix} 0 & -1 & 1 & -1 \\ 7 & 1 & -4 & -9 \\ 2 & 0 & -1 & -3 \\ 1 & 0 & -1 & -1 \end{pmatrix}$$

A.6

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & -1 & -3 & 1 \\ 1 & -1 & -2 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

A.7

$$\begin{pmatrix} 1 & -3 & 2 & 1 \\ -1 & 0 & 0 & 0 \\ -2 & 8 & -5 & -3 \\ -3 & 5 & -3 & -1 \end{pmatrix}$$

A.8

$$\begin{pmatrix} 1 & 5 & 6 & 0 \\ -1 & 0 & 2 & 1 \\ 0 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

A.9

$$\begin{pmatrix} -1 & 1 & 3 & -1 \\ 0 & 2 & 4 & -1 \\ -1 & 1 & 4 & -2 \\ -1 & 0 & 1 & -1 \end{pmatrix}$$

A.10

$$\begin{pmatrix} 0 & -5 & -7 & 7 \\ 0 & -2 & -3 & 3 \\ -1 & 1 & -3 & 0 \\ -1 & 7 & 7 & -9 \end{pmatrix}$$

A.11

$$\begin{pmatrix} 2 & -2 & 0 & 1 \\ 3 & -6 & 0 & 2 \\ -1 & 1 & -1 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}$$