

## 代数学幾何学 (A/B) 計算演習 [問題] (2010/01/14)

問. 次の行列  $A$  によって定まる線型変換  $T_A$  の基底  $E, F$  に関する行列を求めなさい。

Q.1

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ -2 & 0 & 1 & 0 \\ 2 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 \end{pmatrix}$$

$$E = \left\langle \begin{pmatrix} 0 \\ -2 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ -3 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ -2 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} 0 \\ -2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ -1 \\ 2 \end{pmatrix} \right\rangle$$

Q.2

$$A = \begin{pmatrix} -1 & -4 & 0 & 2 \\ 2 & 6 & -1 & -3 \\ 0 & 3 & 1 & -2 \\ -2 & -5 & 1 & 3 \end{pmatrix}$$

$$E = \left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ -1 \end{pmatrix} \right\rangle$$

Q.3

$$A = \begin{pmatrix} 3 & -1 & 0 & -1 \\ 5 & 4 & -6 & 2 \\ 2 & 2 & -3 & 1 \\ -1 & -2 & 2 & -1 \end{pmatrix}$$

$$E = \left\langle \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ -1 \\ -1 \end{pmatrix} \right\rangle$$

$$F = \left\langle \begin{pmatrix} 0 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ -2 \\ -3 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\rangle$$

Q.4

$$A = \begin{pmatrix} -1 & -3 & 2 & 4 \\ 0 & -2 & 1 & 2 \\ -2 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} 3 \\ 3 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \\ 3 \\ -4 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \\ -2 \\ 1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} -5 \\ 5 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ -5 \\ -5 \\ -2 \end{pmatrix}, \begin{pmatrix} 4 \\ -4 \\ -2 \\ -1 \end{pmatrix} \right\rangle$$

Q.5

$$A = \begin{pmatrix} -7 & -3 & 4 & 2 \\ -5 & -2 & 3 & 1 \\ 1 & 0 & -1 & 2 \\ -1 & 0 & 2 & -3 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -3 \\ -4 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \\ -2 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 2 \\ 3 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} -4 \\ -5 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -6 \\ -6 \\ -4 \\ 1 \end{pmatrix} \right\rangle$$

Q.6

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ -1 & 3 & 1 & 4 \\ 1 & -2 & -1 & -3 \\ -2 & -3 & 1 & -2 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} 4 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -3 \\ -4 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ -1 \\ 3 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} -2 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

Q.7

$$A = \begin{pmatrix} 1 & -7 & 5 & 1 \\ -2 & 7 & -7 & 0 \\ 1 & -3 & 3 & 0 \\ -1 & 1 & -4 & 2 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} -1 \\ 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ -2 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

Q.8

$$A = \begin{pmatrix} -1 & 3 & 2 & 0 \\ 2 & -4 & -4 & -1 \\ -1 & 4 & 2 & -1 \\ 1 & -4 & -3 & 0 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} -1 \\ -1 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -4 \\ -5 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 5 \\ 9 \\ -12 \\ -9 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -5 \\ -4 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ -1 \end{pmatrix} \right\rangle$$

Q.9

$$A = \begin{pmatrix} -12 & -7 & 1 & 15 \\ 5 & 2 & -2 & -7 \\ -2 & -2 & -2 & 1 \\ 5 & 3 & 0 & -6 \end{pmatrix}$$
$$E = \left\langle \begin{pmatrix} -2 \\ -1 \\ -3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 5 \\ -2 \end{pmatrix} \right\rangle$$
$$F = \left\langle \begin{pmatrix} 2 \\ 5 \\ 5 \\ 1 \end{pmatrix}, \begin{pmatrix} -5 \\ -9 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -4 \\ -1 \end{pmatrix} \right\rangle$$

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A.1

$$\begin{pmatrix} -5 & -7 & -5 & 2 \\ 1 & 2 & 3 & -1 \\ 5 & 9 & 4 & -2 \\ 4 & 7 & 5 & -2 \end{pmatrix}$$

A.2

$$\begin{pmatrix} -7 & -16 & 12 & 4 \\ 44 & 104 & -79 & -28 \\ -25 & -59 & 45 & 16 \\ 18 & 42 & -32 & -11 \end{pmatrix}$$

A.3

$$\begin{pmatrix} -3 & -17 & 1 & -15 \\ -7 & 26 & 23 & -3 \\ 7 & 9 & -12 & 20 \\ -9 & -10 & 16 & -25 \end{pmatrix}$$

A.4

$$\begin{pmatrix} -207 & 59 & -347 & 101 \\ 27 & -8 & 46 & -13 \\ -183 & 52 & -307 & 89 \\ 161 & -46 & 271 & -78 \end{pmatrix}$$

A.5

$$\begin{pmatrix} 23 & -58 & 90 & -49 \\ 41 & -104 & 150 & -82 \\ -14 & 36 & -47 & 26 \\ -24 & 61 & -84 & 46 \end{pmatrix}$$

A.6

$$\begin{pmatrix} 10 & 3 & 30 & -43 \\ -23 & -7 & 13 & 1 \\ -3 & -1 & 22 & -24 \\ -12 & -4 & -19 & 32 \end{pmatrix}$$

A.7

$$\begin{pmatrix} -69 & 44 & 33 & 47 \\ -35 & 19 & 15 & 20 \\ 63 & -36 & -28 & -38 \\ 123 & -71 & -55 & -75 \end{pmatrix}$$

A.8

$$\begin{pmatrix} -34 & 25 & -17 & 1 \\ 1 & 12 & -2 & -9 \\ -96 & 86 & -51 & -8 \\ 20 & -20 & 11 & 3 \end{pmatrix}$$

A.9

$$\begin{pmatrix} 225 & 100 & 58 & -294 \\ 614 & 276 & 170 & -793 \\ 1339 & 603 & 375 & -1726 \\ 975 & 438 & 269 & -1260 \end{pmatrix}$$